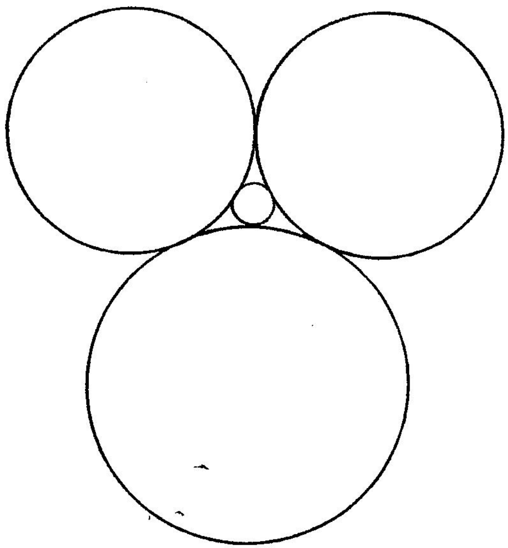
Honors Geometry

Name____________________

Find the coordinates of the centroid, circumcenter, and the orthocenter for a triangle EDY with points E(-4,0), D(0,5), Y(10,-3). Show that these three points are collinear then find the equation of this line. This is called the Euler line for triangle EDY.

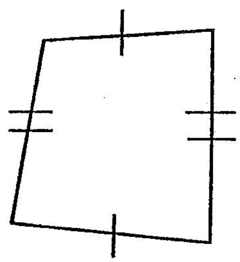
Honors Geometry
Group Problem

Circles of radius 5, 5, 8 and m/n are mutually externally tangent, where m and n are relatively prime positive integers. Find $m+n$.

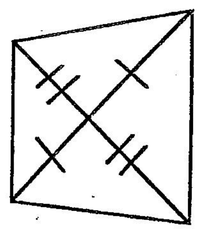


What is the most specific name for each quadrilateral?

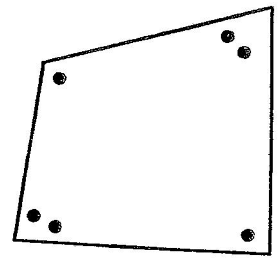
1)



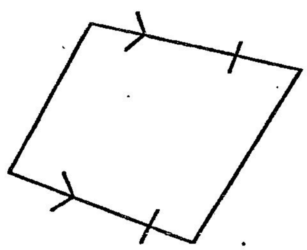
2)



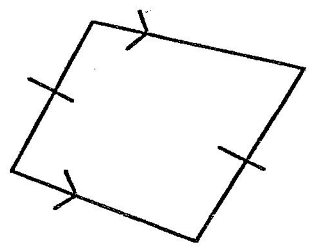
3)



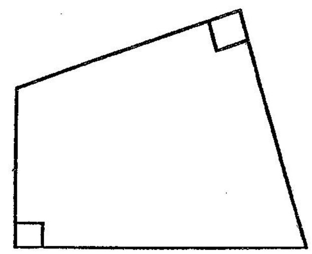
4)



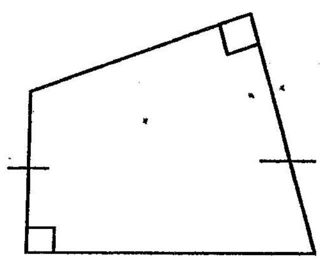
5)



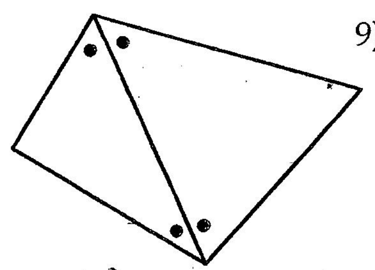
6)



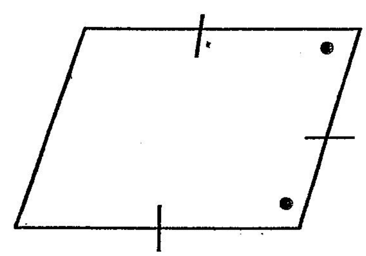
7)



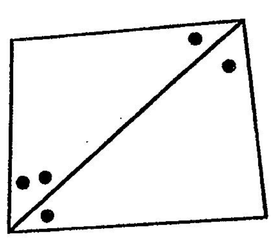
8)



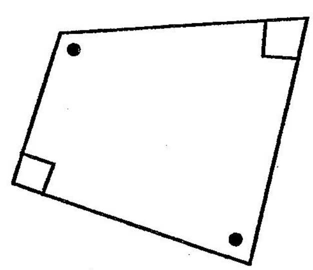
9)



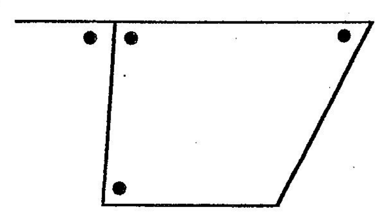
10)



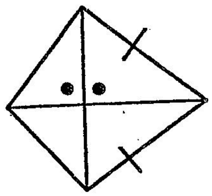
11)



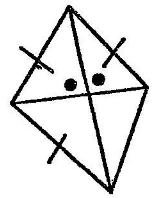
12)



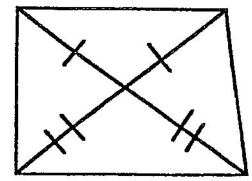
13)



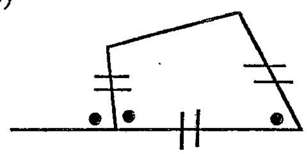
14)



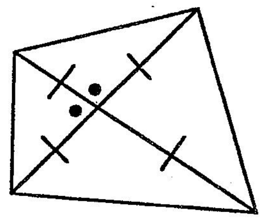
15)



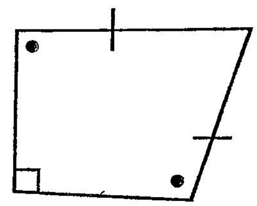
16)



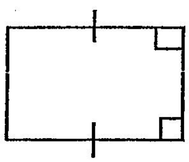
17)



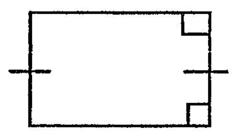
18)



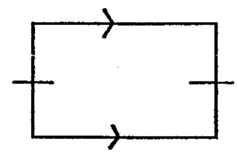
19)



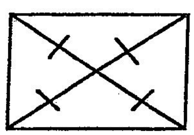
20)



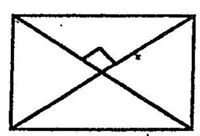
21)



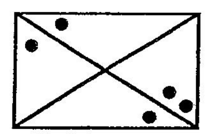
22)



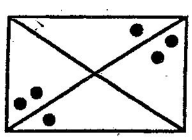
23)



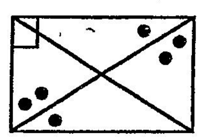
24)



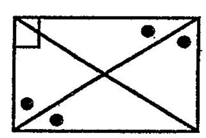
25)



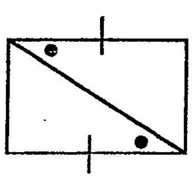
26)



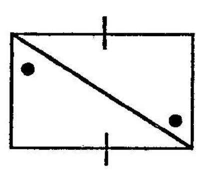
27)



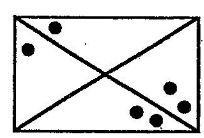
28)



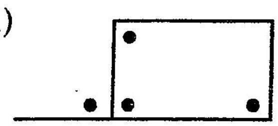
29)



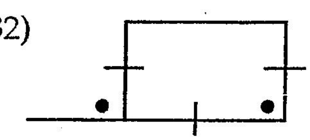
30)



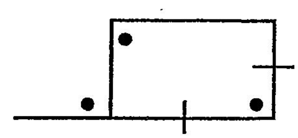
31)



32)

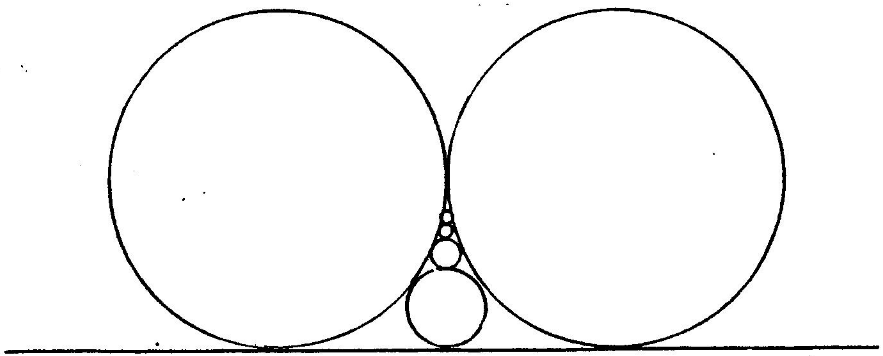


33)



Geometry Honors

Each of the two large circles below have radius 1, and are tangent to each other and the line. A sequence of smaller circles, each having the largest possible radius, are drawn as shown, each being tangent to the line and/or circles within which they are inscribed. Find the diameters of each of the smaller circles (How many of the smaller circles will there be?). What is the sum of their diameters?



Geometry Honors
Group Problem

```
        G
      E   E
    O   O   O
  M   M   M   M
E   E   E   E   E
  T   T   T   T
    R   R   R
      I   I
        C
```

1. How many paths can you take starting at G which pass through the middle O and which spell the word GEOMETRIC? In each step you are allowed to move to the letter at the immediate lower right or lower left.

2. How many paths through the left O?

3. Through any O?

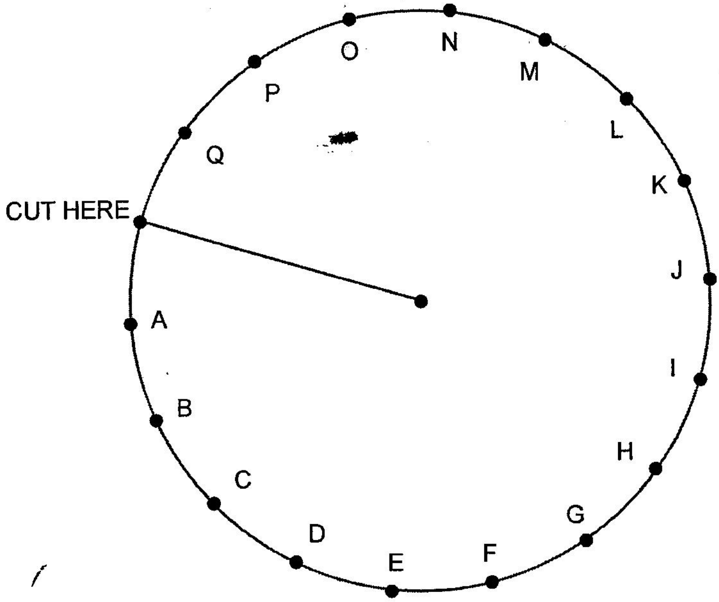
4. If a path is chosen at random, what is the probability it passes through the leftmost M?

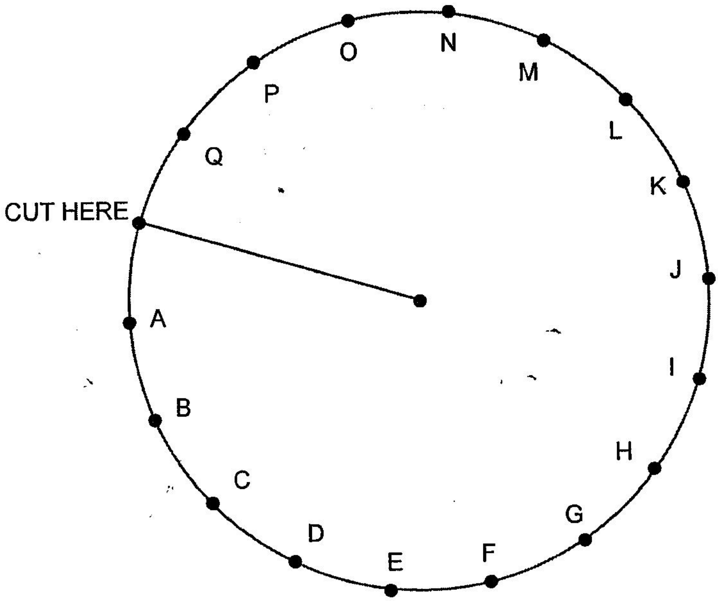
5. What is the probability a path passes through one of the two central M's?

Geometry Honors
Nets for cones

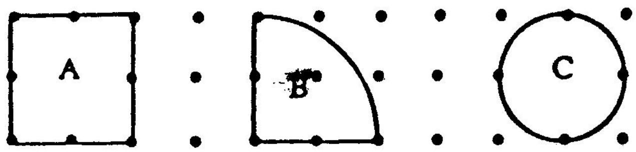
Use the template provided to create cones of various bases and heights. Find the total surface areas of at least three different cones. Find a formula relating the central angle with surface area.

What central angle do you think will create the greatest volume? Find the volume of each of the cones created, and find a formula relating the central angle to volume. What central angle gives the most volume?


O
N
M
P
L
Q
K
CUT HERE
J
A
I
B
H
C
G
D
F
E

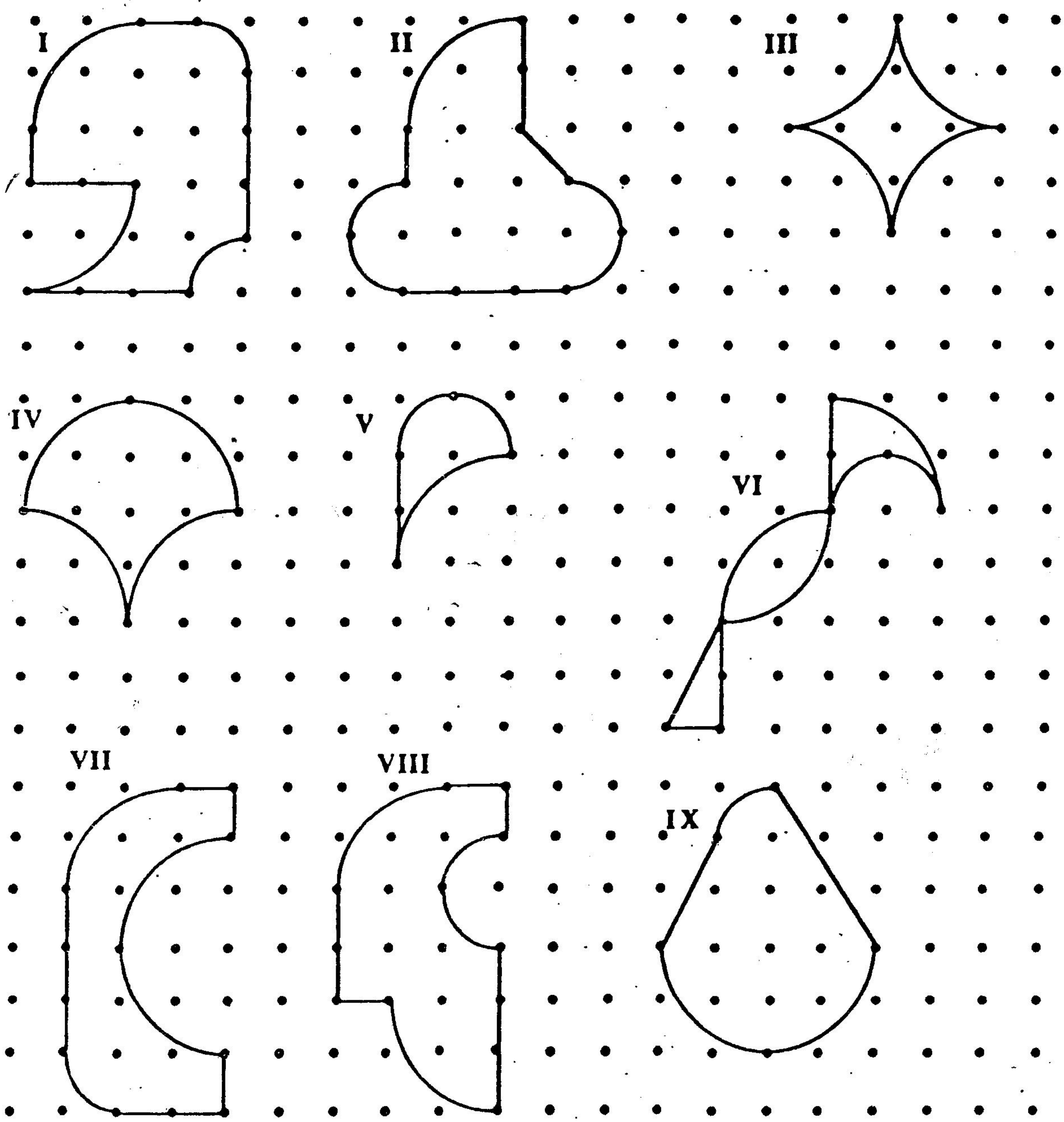

O
N
M
P
L
Q
K
CUT HERE
J
A
I
B
H
C
G
D
F
E

AREA ASSIGNMENT



The area of A is a and
the area of B is b and
the area of C is b.

Find the areas of the following in terms of a and b.

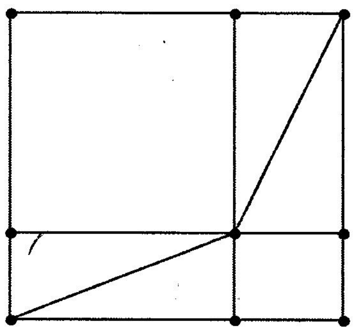


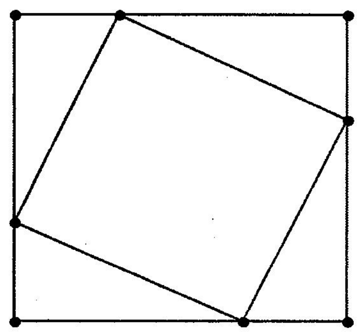
(Abacus is a portion of the Ontario Math Gazette published at Univ. of Western Ont.)

Honors Geometry
Pythagorean Theorem

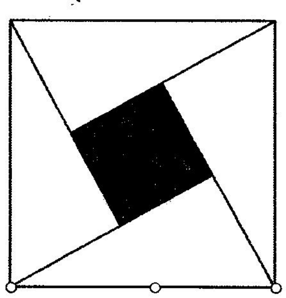
Below are two visual proofs of the Pythagorean Theorem. Explain how the relationship between the figure on the left to the figure in the right is a proof of "Geometry's Most Elegant Theorem."

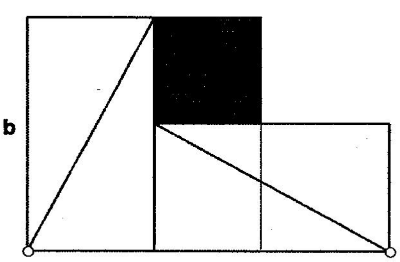
The following is a proof from China about 200 B.C. Explain how it works.



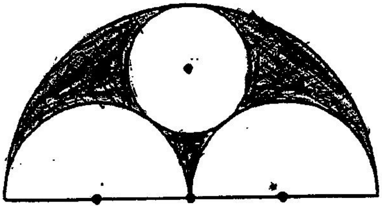


This one requires some algebra!!! The following is a proof of the Pythagorean Theorem drawn by Bhaskara Acharya (114-1185), a Hindu mathematician. It was accompanied by the word- "BEHOLD!" Explain why it works!!!





Two congruent semicircles lie on the diameter of a third semicircle, each tangent to the other two. A small circle is tangent to all three semicircles. If the area of the shaded region is 45π, what is the length of a radius of the small circle?



Geometry Honors
Pythagorean Triples

Here are some Primitive Pythagorean Triples (PPTs) we discussed in class:
(3, 4, 5), (8, 15, 17), (9, 40, 41), (7, 24, 25), (11, 60, 61), (5, 12, 13), (12, 35, 37), (20, 21, 29)

Here are some conjectures we made in class. Try to see if you can explain (or disprove!) any of these as you experiment further.

- Every PPT contains a prime number.
- Every PPT contains a number divisible by three. (Also true for four and five).
- Every PPT contains a pair of numbers within two of each other.
- Every PPT contains exactly two odd numbers.
- There are an infinite number of PPTs.

Examine the differences between consecutive squares. What is the difference between n^2 and $(n+1)^2$

Find 5 more PPTs Do our conjectures hold for your new PPTs?

Is there a PPT with the number 31? If so find one. If not, explain how you know.

Answer the following questions for all of the numbers 1-100. What patterns do you notice? Can you make any new conjectures? You might want to make a chart with your PPTs, and their related PTs as reference, and write out your answers on separate paper.

Which numbers can be part of a PPT and which numbers cannot? Which numbers are in more than one PPT? Which numbers are in more than one PT?

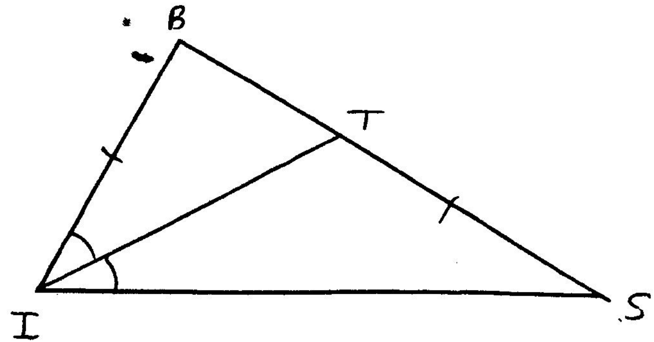
One-hundred Perfect Squares

N	N^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400
21	441
22	484
23	529
24	576
25	625
26	676
27	729
28	784
29	841
30	900
31	961
32	1024
33	1089
34	1156
35	1225
36	1296
37	1369
38	1444
39	1521
40	1600
41	1681
42	1764
43	1849
44	1936
45	2025
46	2116
47	2209
48	2304
49	2401
50	2500
51	2601
52	2704
53	2809
54	2916
55	3025
56	3136
57	3249
58	3364
59	3481
60	3600
61	3721
62	3844
63	3969
64	4096
65	4225
66	4356
67	4489
68	4624
69	4761
70	4900
71	5041
72	5184
73	5329
74	5476
75	5625
76	5776
77	5929
78	6084
79	6241
80	6400
81	6561
82	6724
83	6889
84	7056
85	7225
86	7396
87	7569
88	7744
89	7921
90	8100
91	8281
92	8464
93	8649
94	8836
95	9025
96	9216
97	9409
98	9604
99	9801
100	10000

Geometry Honors

In Δ IBS, $\overline{IB} \cong \overline{TS}$

BT + IS = 30, $\overline{IT}$ bis $\angle$BIS and all the sides of the Δ are whole numbers. Find all the possible perimeters of Δ IBS. Explain how you arrived at your results.



Geometry Honors

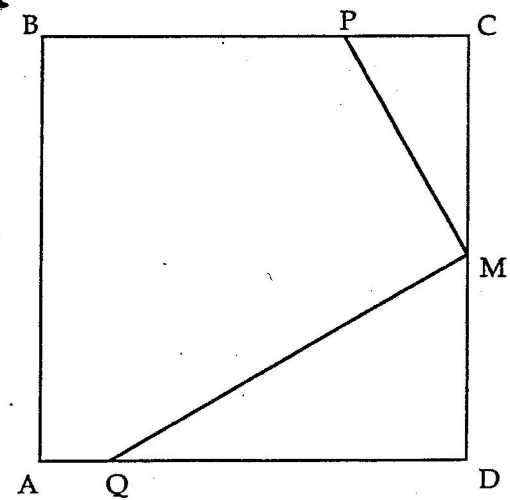
Group Problem

Given: ABCD is a square

$AB = 1$, M mid $\overline{CD}$

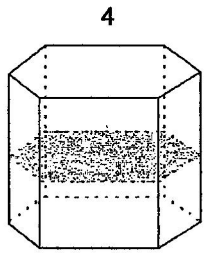
$\overline{QM} \perp \overline{PM}$

Find: $(PC)(QD)$

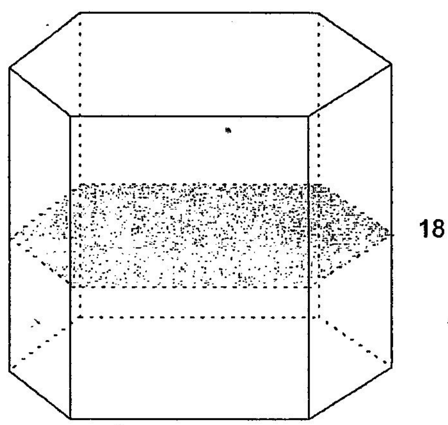


Geometry Honors
Ratios of areas and volumes

1. Find the surface area, volume, and area of the indicated cross-section of the following regular hexagonal prism.



2. Now find the surface area, volume, and area of the indicated cross-section of this prism. [Note: This prism is similar to the prism in problem 1 with a scale ratio of 1:3.]



3. How many cubes of sidelength 1 can fit inside a cube of sidelength 3? What is the ratio of surface area of these two cubes?

4. Complete the following sentence: If two solids are similar with scale ratio r : s, their surface areas are in a ratio ______: ______, and their volumes are in a ratio _________ : ___________.

5. Two spheres' volumes are in a ratio of 20: 50. Find the ratios of their areas and radii.

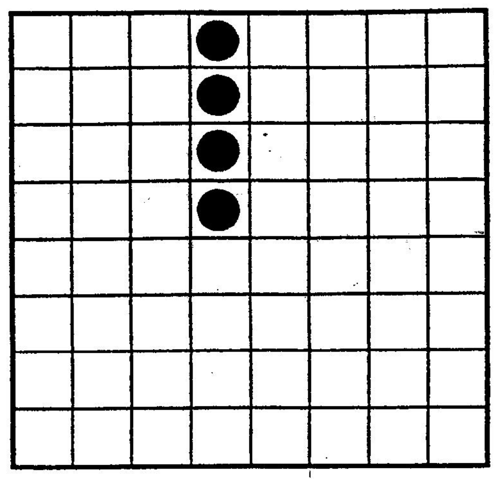
6. Assume a 6-foot person has a surface area: volume ratio of 10 : 20. Assuming a 4-foot tall pygmy is similar to the 6-foot person, what would the surface area : volume ratio of the pygmy be?

7. A soup company wants to minimize the surface area : volume ratio of its cylindrical $75cm^3$ soup cans. Why would they want to do this? What should the dimensions of the cans be?

8. The strength of a muscle is proportional to its cross-sectional area, while an animal's weight is proportional to its volume. How can you use these facts to explain why a flea can jump hundreds of times its own height, but a person can't even jump his or her own height (and an elephant has trouble getting off the ground!)

Geometry Honors
Group Problem

Divide the figure into four congruent parts each of which contain one of the dots. You may only divide along the gridlines. Furthermore, the dots may not be in the same position in each of the parts.



Honors Geometry
More on the Platonic Solids!!!
Name:

1) What figure will you get if you connect the center of gravity of each face of a tetrahedron?

2) Fill in the table below:

	Tetrahedron	Octahedron	Icosahedron	Cube	Dodecahedron
Number of faces					
Number of Edges					
Number of vertices					
Number of faces meeting at each vertex					

3) Do you notice any pattern with the numbers in each column?

4) Do you notice any similarities in the numbers of different columns?

5) What figure will you get if you connect the center of gravity of each face of a cube?

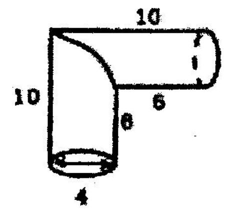
6) What figure will you get if you connect the center of gravity of each face of the figure that you got in question 5? Why do you think that this will happen?

7) What figure will you get if you connect the center of gravity of each face of the dodecahedron? Why?

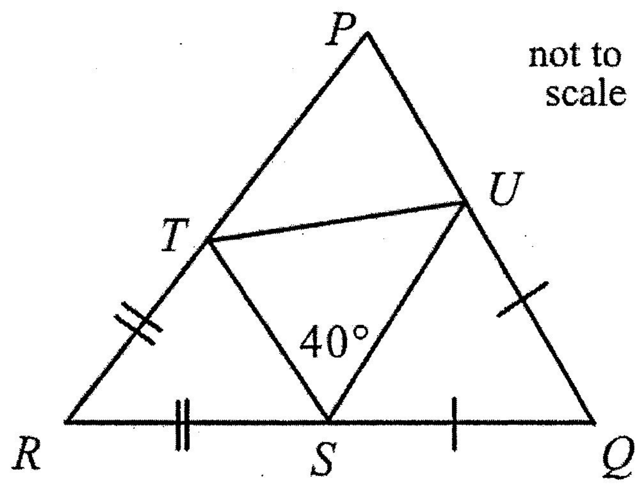
8) Why do you think that your answer to #1 was what it was?

Ten+ Final Review Problems

1. This L-shaped pipe connector is designed to join two drain pipes together so water can go around a corner. The connector consists of two overlapping cylinders with the same radius. Find the total volume.



2.



not to scale

Find the measure of angle P.

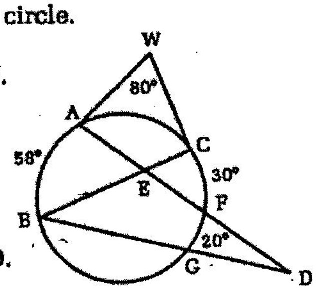
3. $\overline{WA}$ and $\overline{WC}$ are tangent to the circle.

1 Find the measure of $\angle CEF$.

2 Find the measure of $\angle D$.

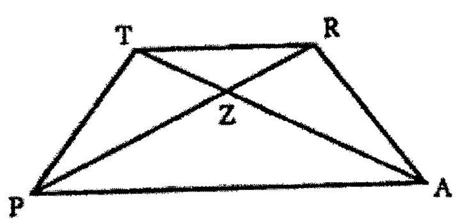
3 Find the measure of $\overset{\frown}{AC}$.

4 Find the measure of $\angle EBD$.



5 Find the circumference of a circle in which an 18-cm chord is 40 cm from the center.

4. **TRAP is an isosceles trapezoid with bases $\overline{TR}$ and $\overline{PA}$. $m\angle TPA = 70$, $m\angle RAZ = 5x - 8$, $m\angle ZAP = 2x + 1$, $PR = x + 2$.**



Find *x* and TA.

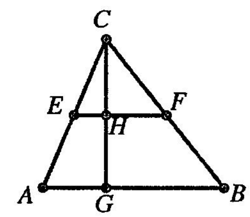
5. As a spherical gob of ice cream that once had a 2-inch radius melts, it drips into a right circular cone of the same radius. The melted ice cream exactly fills the cone. What is the height of the cone?

6. Is it possible for $\sin\theta$ to be exactly twice the size of $\cos\theta$? If so, find such an angle θ. If not, explain why not.

7. Prove that if the median to the hypotenuse of a right triangle is perpendicular to the hypotenuse, then the triangle is isosceles.

8. One diagonal of a certain quadrilateral bisects two angles of the quadrilateral. Prove that it bisects the other diagonal.

9. In triangle ABC (shown on the right), segment CG is the altitude to side AB, line EHF is parallel to side AB, and the area of triangle CEF is half the area of triangle ABC. If CG = 10, find CH.



10 Consider the points W(0, 3); X(6, 4); Y(12, –3); Z(–2, –12).

   a. Which two lines determined by these points are perpendicular?
   b. Find the exact distances WX and XY.
   c. What kind of figure is WXYZ? Be as specific as possible.

Also, in the textbook, p. 598, #1–9, 11, 13–23, 25–42.

(assembled from various sources)