

Geometry Review

“Say you’re me and you’re in math class....”



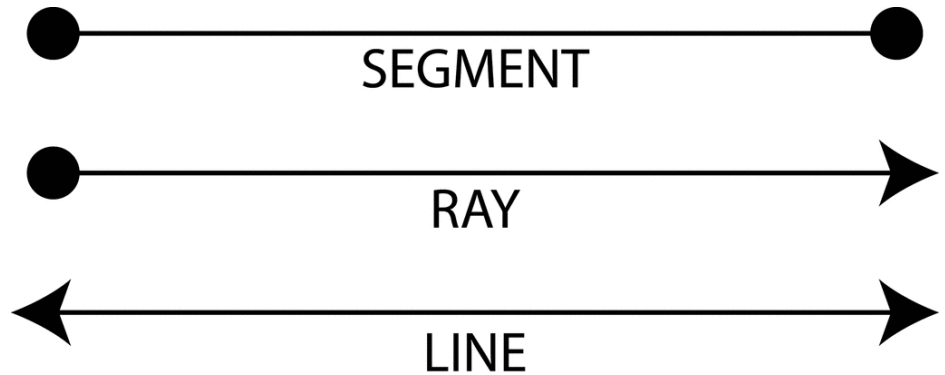
Geometry Cohort

Weston Middle School

June 2013

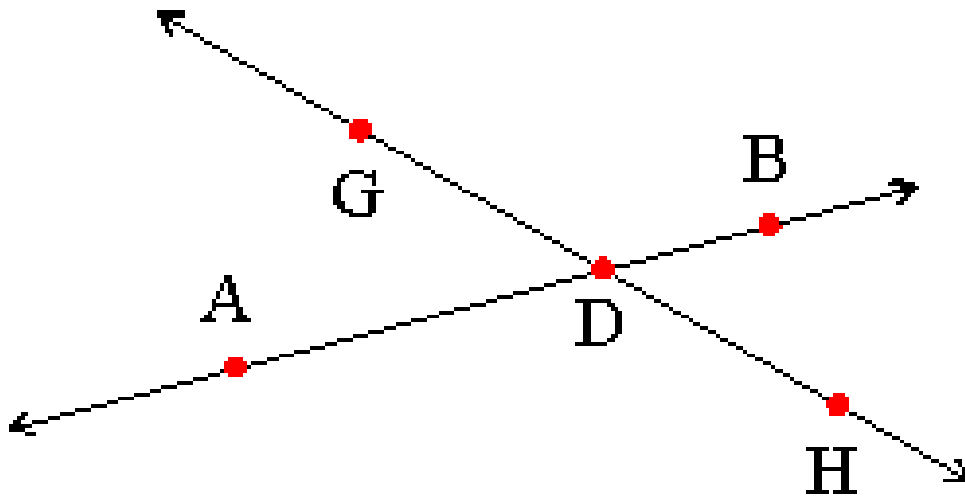
Definitions

- Point—0 Dimensions- P
- Line Segment AB
- Midpoint
- Ray



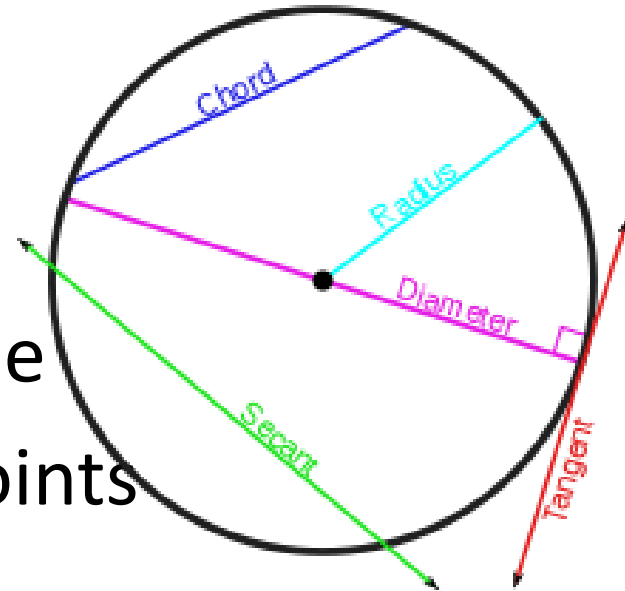
Definitions

- 2 Co-linear Points define a line
- 2 intersecting lines define a plane



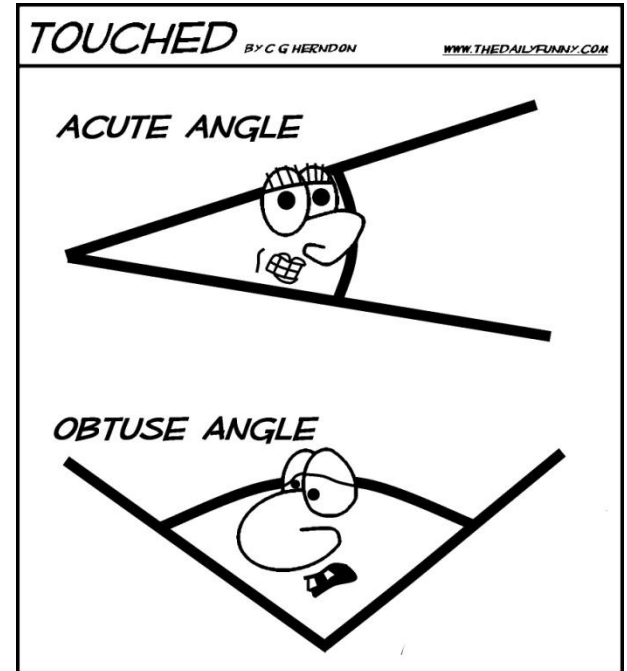
Circle Definitions

- Circle
- Radius
- Chord- 2 points on a circle
- Secant- line through 2 points
- Diameter
- Tangent
- Major Arc, Minor Arc



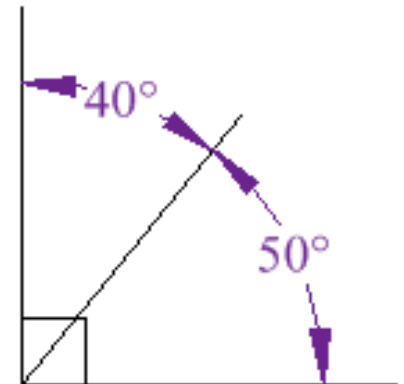
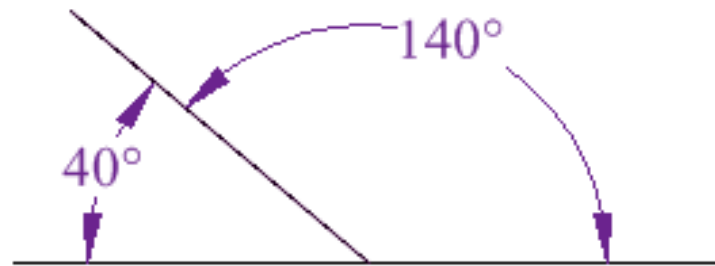
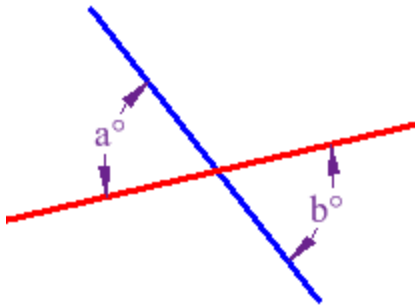
Angles

- 2 rays form angle
- Vertex, Sides
- Adjacent Angles-share a side
- Acute Angle- < 90
- Right Angles
- Obtuse Angles > 90
- Straight angle = 180
- Reflex Angle > 180



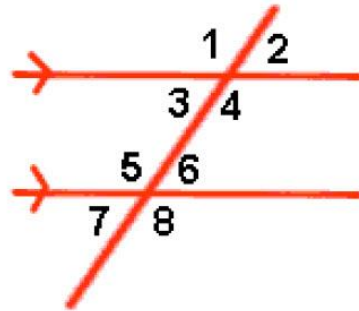
Angles-2

- Lines that intersect form Vertical angles
- Vertical angles are equal
- Supplementary angles add up to 180
- Complementary angles add up to 90



Parallel Lines and Transversals

Parallels: If lines are parallel ...



Corresponding angles are equal.

$$m\angle 1 = m\angle 5, m\angle 2 = m\angle 6, m\angle 3 = m\angle 7, m\angle 4 = m\angle 8$$

Alternate Interior angles are equal.

$$m\angle 3 = m\angle 6, m\angle 4 = m\angle 5$$

Alternate Exterior angles are equal.

$$m\angle 1 = m\angle 8, m\angle 2 = m\angle 7$$

Same side interior angles are supp.

$$m\angle 3 + m\angle 5 = 180, m\angle 4 + m\angle 6 = 180$$

Triangle Types

Triangles:

By Sides:

Scalene – no congruent sides

Isosceles – 2 congruent sides

Equilateral – 3 congruent sides

By Angles:

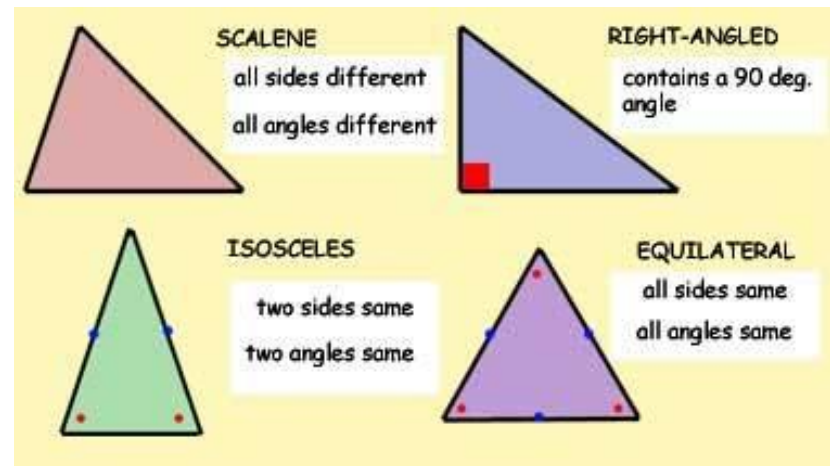
Acute – all acute angles

Right – one right angle

Obtuse – one obtuse angle

Equiangular – 3 congruent angles(60°)

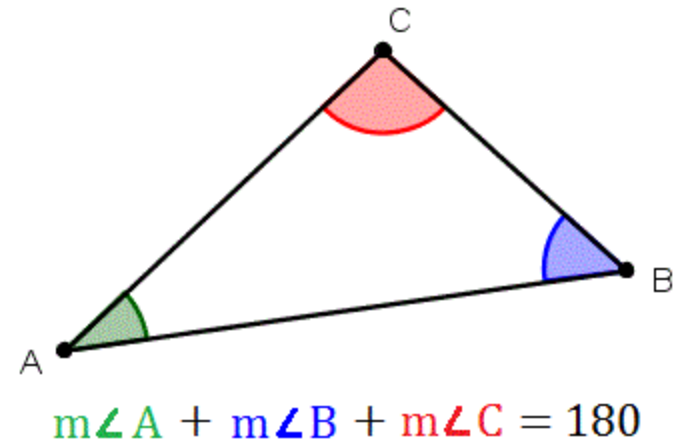
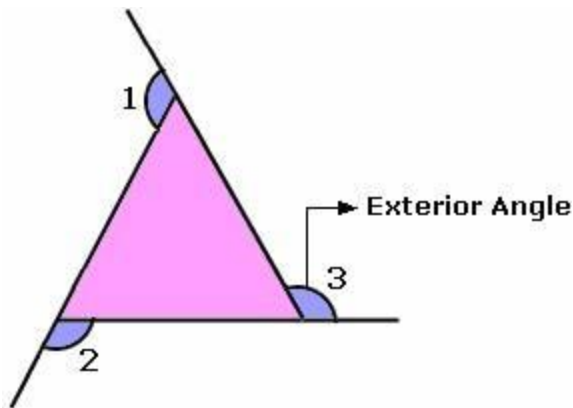
Equilateral \leftrightarrow Equiangular



Exterior angle of a triangle equals the sum of the 2 non-adjacent interior angles.

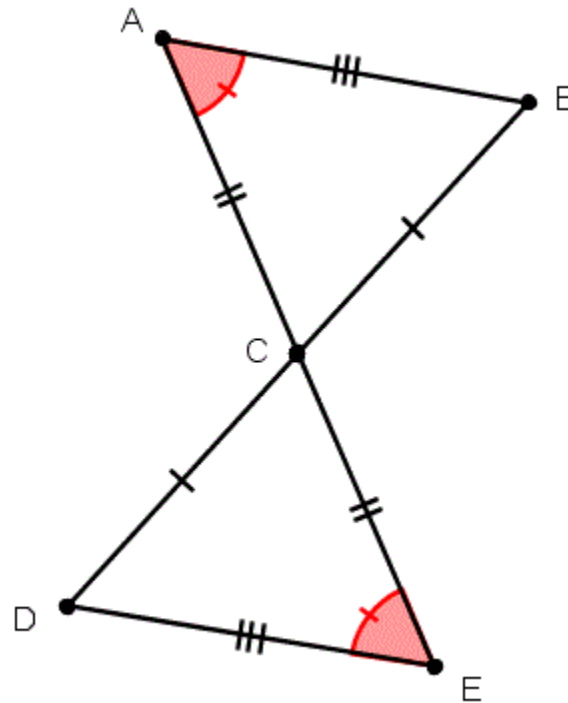
Triangles-1

- 3 points connected with line segments form triangle
- Sum of interior angles= 180 degrees
- Exterior angle = sum of its remote interior angles



Congruent Triangles

Two figures are congruent if they can sit on top of each other



Congruent Triangles

Congruent Triangles

SSS

SAS

ASA

AAS

HL (right triangles only)

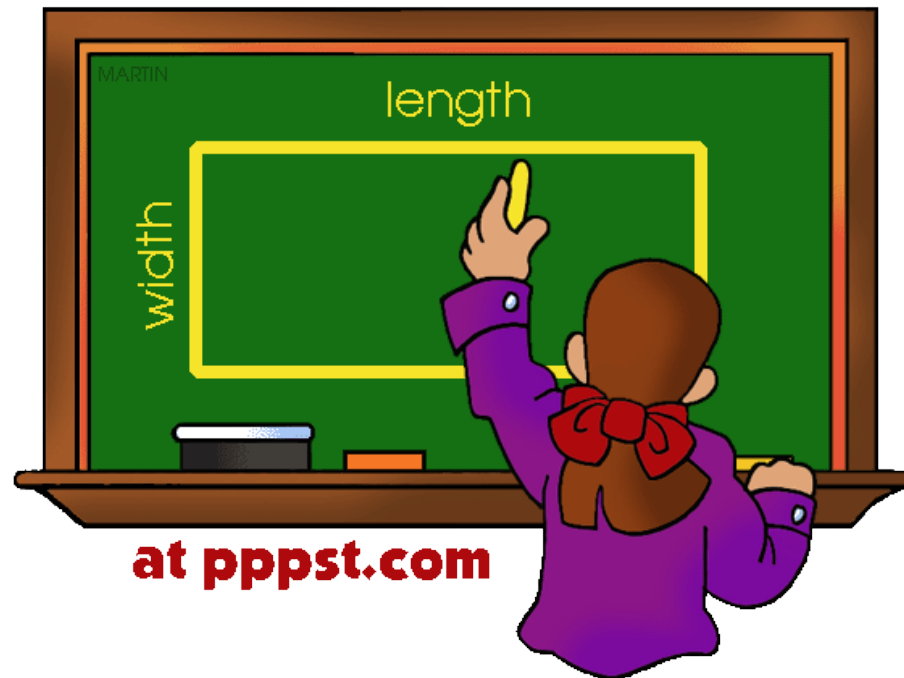
NO donkey theorem
(SSA or ASS)

CPCTC (use after the triangles are congruent)

Perimeter, Area

- Perimeter= measurement around
- Area

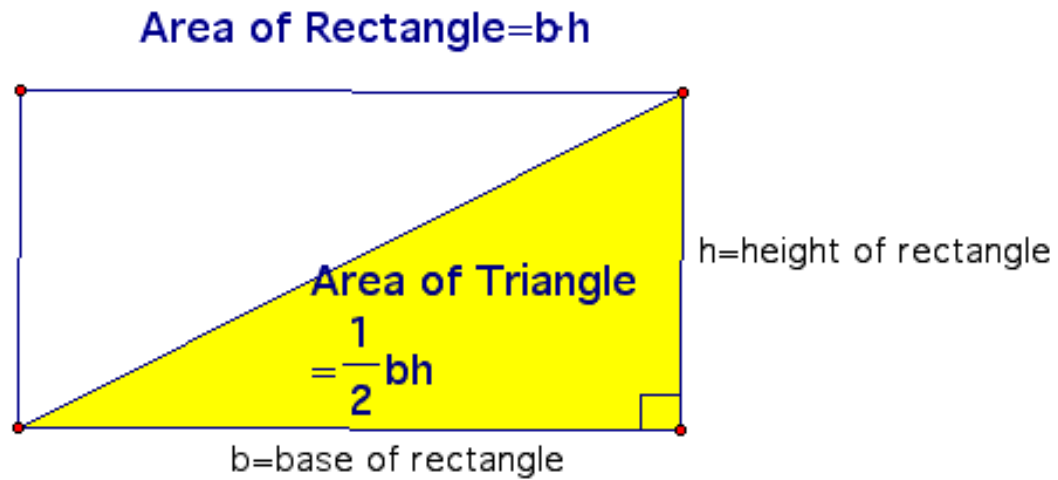
AREA and PERIMETER



Rectangle- 4 right angles, opposite sides equal

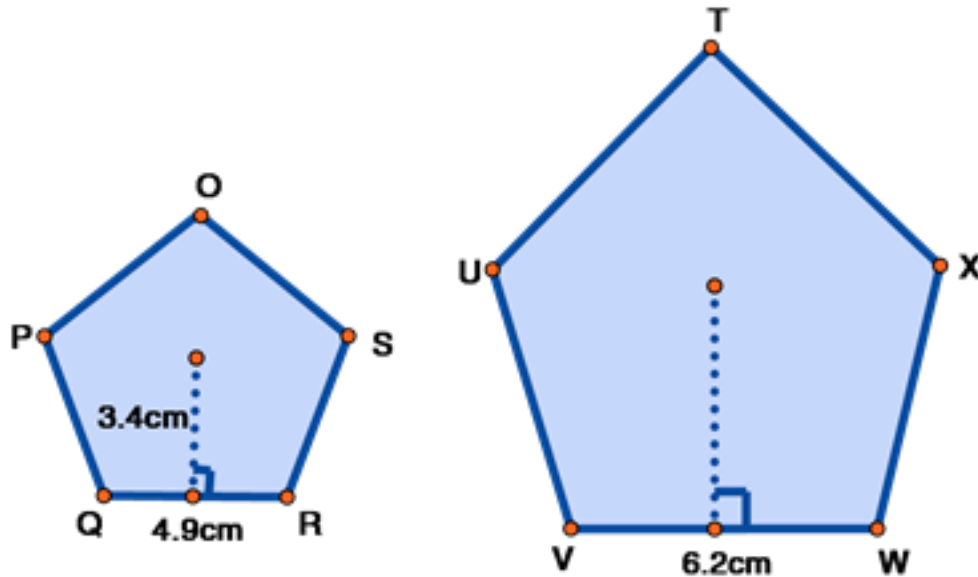
Right Triangle- has 2 legs and a hypotenuse

Area of right triangle= $\frac{1}{2}$ product of legs



Similarity

- Two figures are similar if one is simply a blown-up/rotated version of the other



Similar Triangles

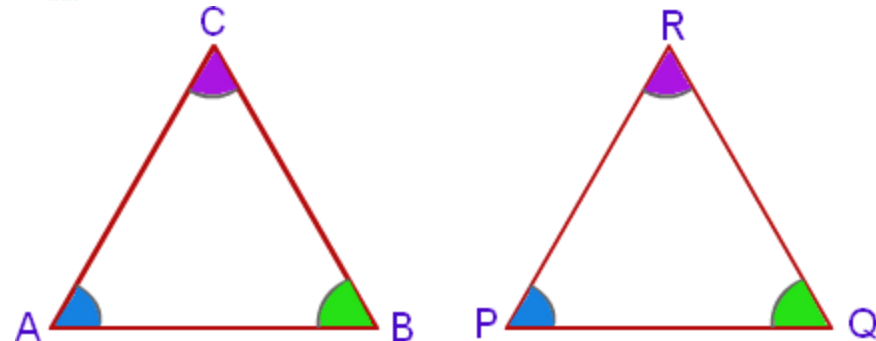
Similar Triangles:

AA

SSS for similarity

SAS for similarity

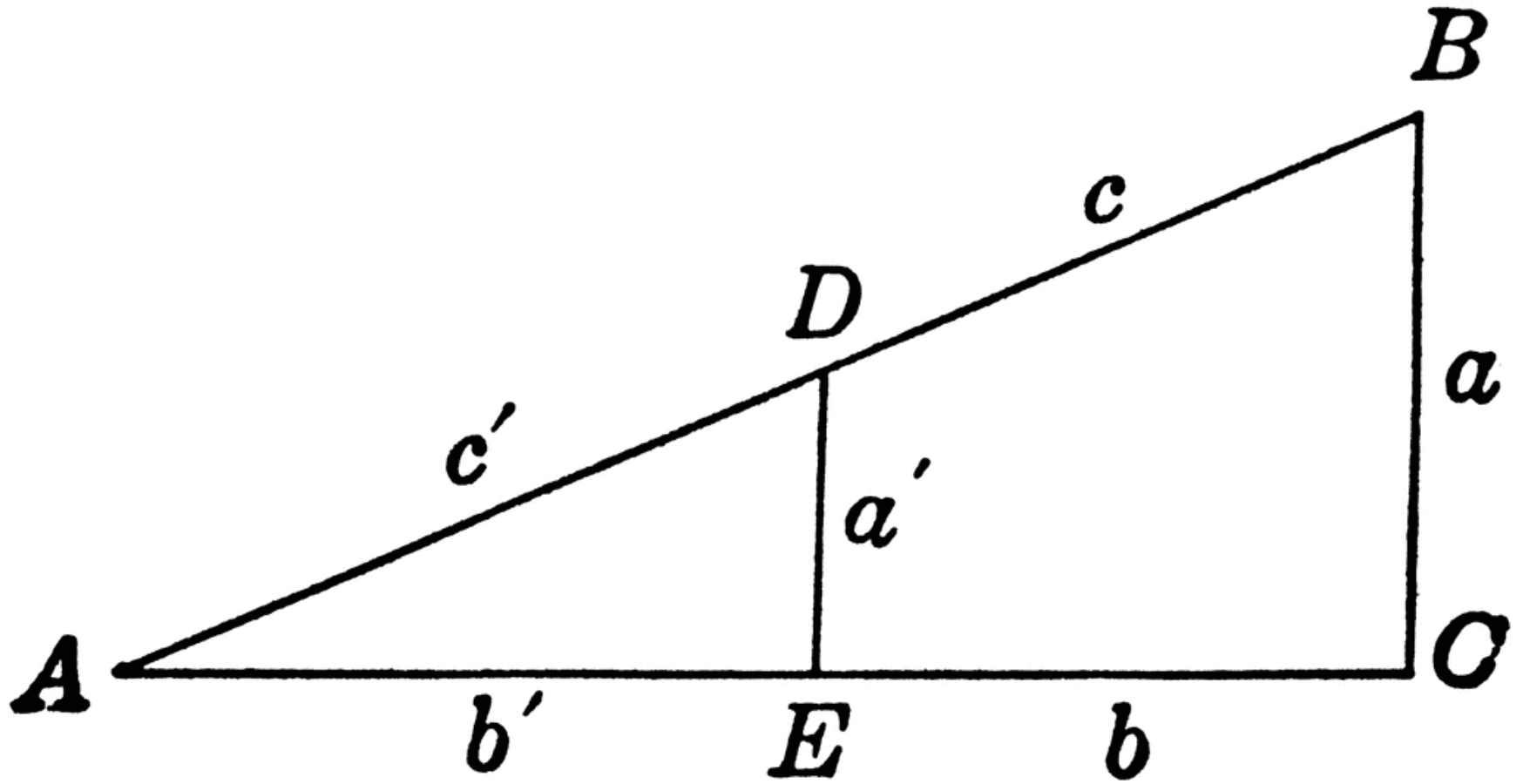
Corresponding sides of similar triangles are in proportion.



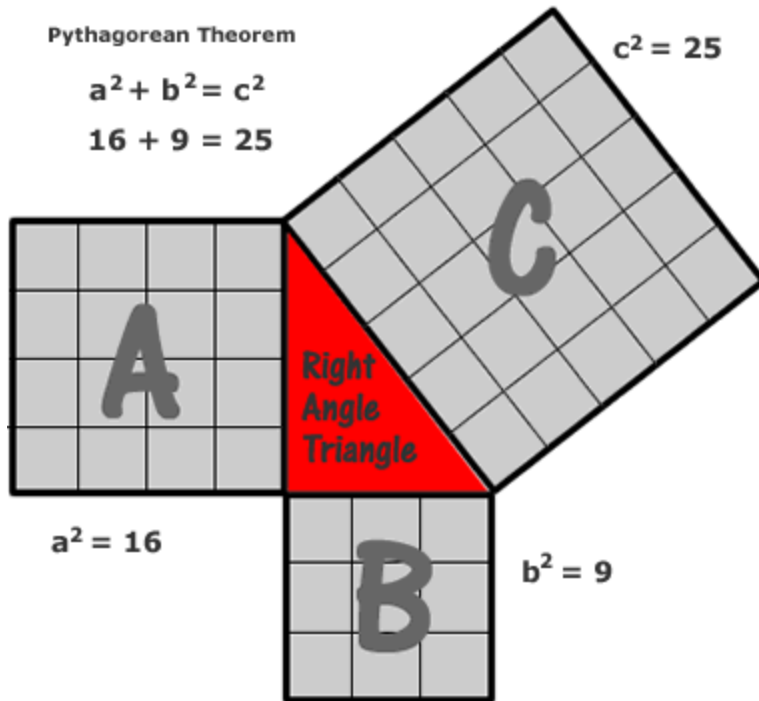
$$\triangle ABC \sim \triangle PQR$$

$$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$$

Similar Triangles



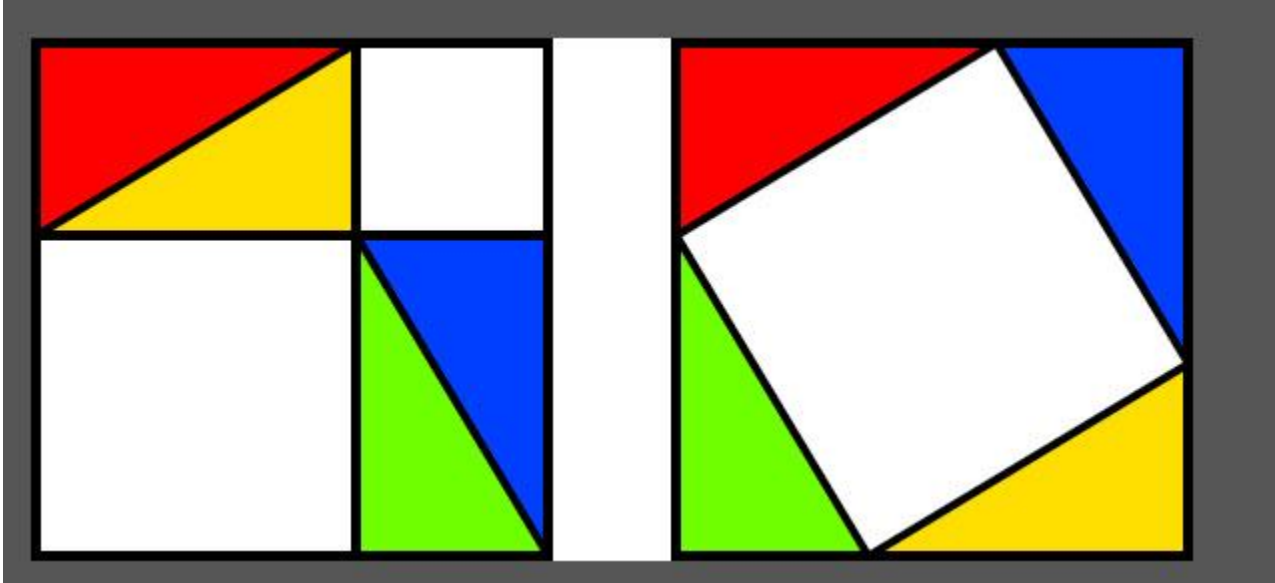
Right Triangles



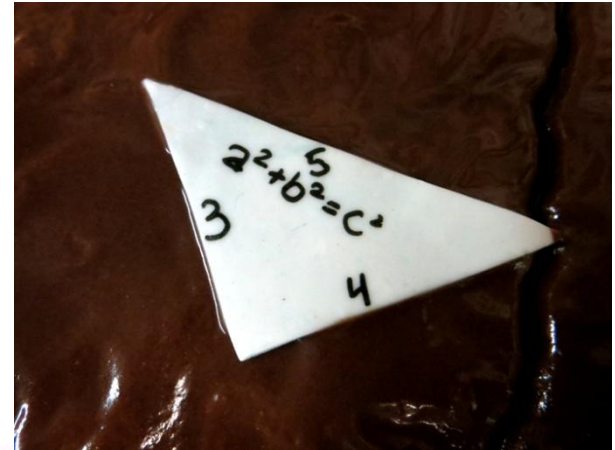
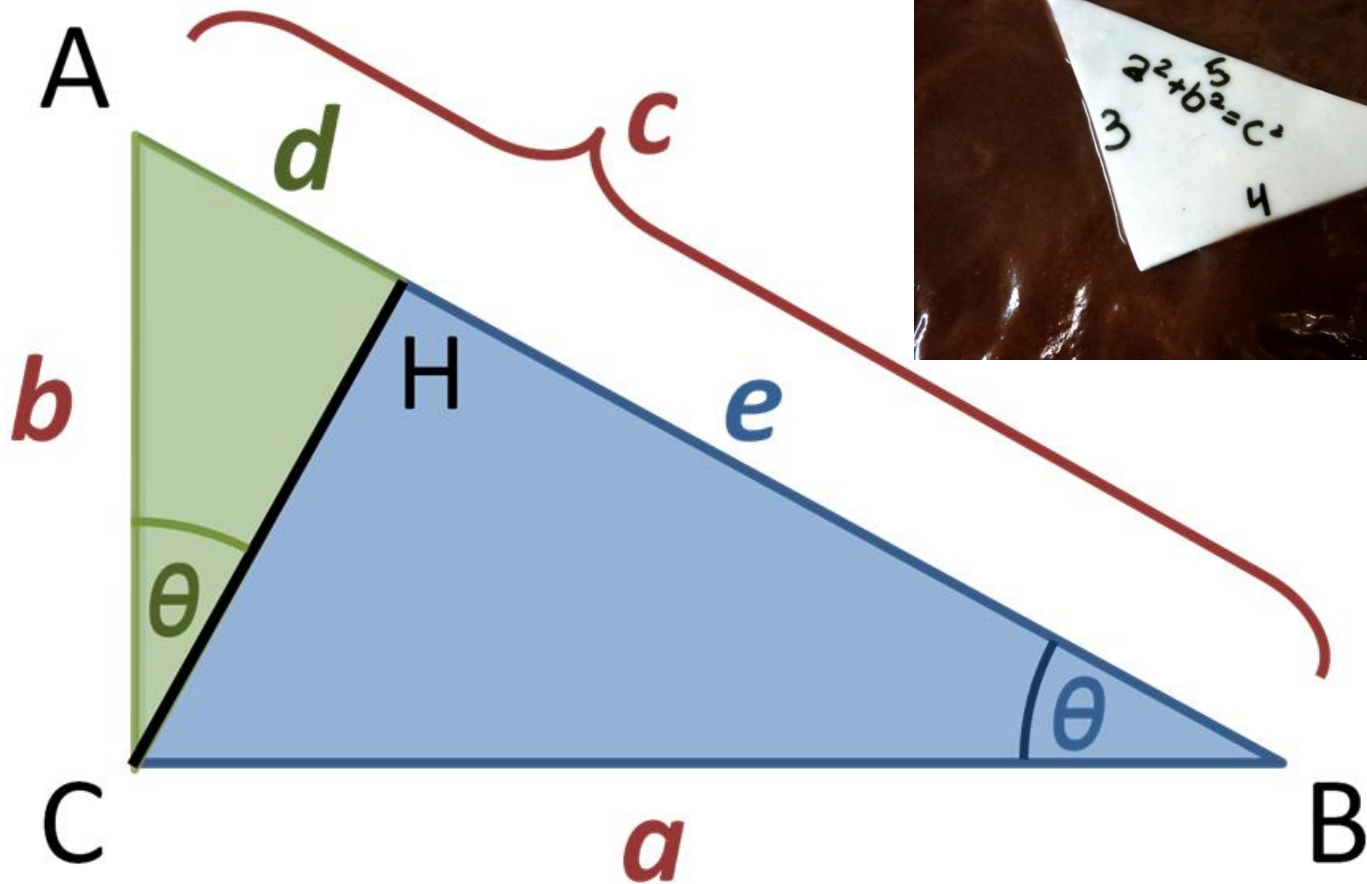
Pythagorean Theorem:

$$c^2 = a^2 + b^2$$

Converse: If the sides of a triangle satisfy $c^2 = a^2 + b^2$ then the triangle is a right triangle.

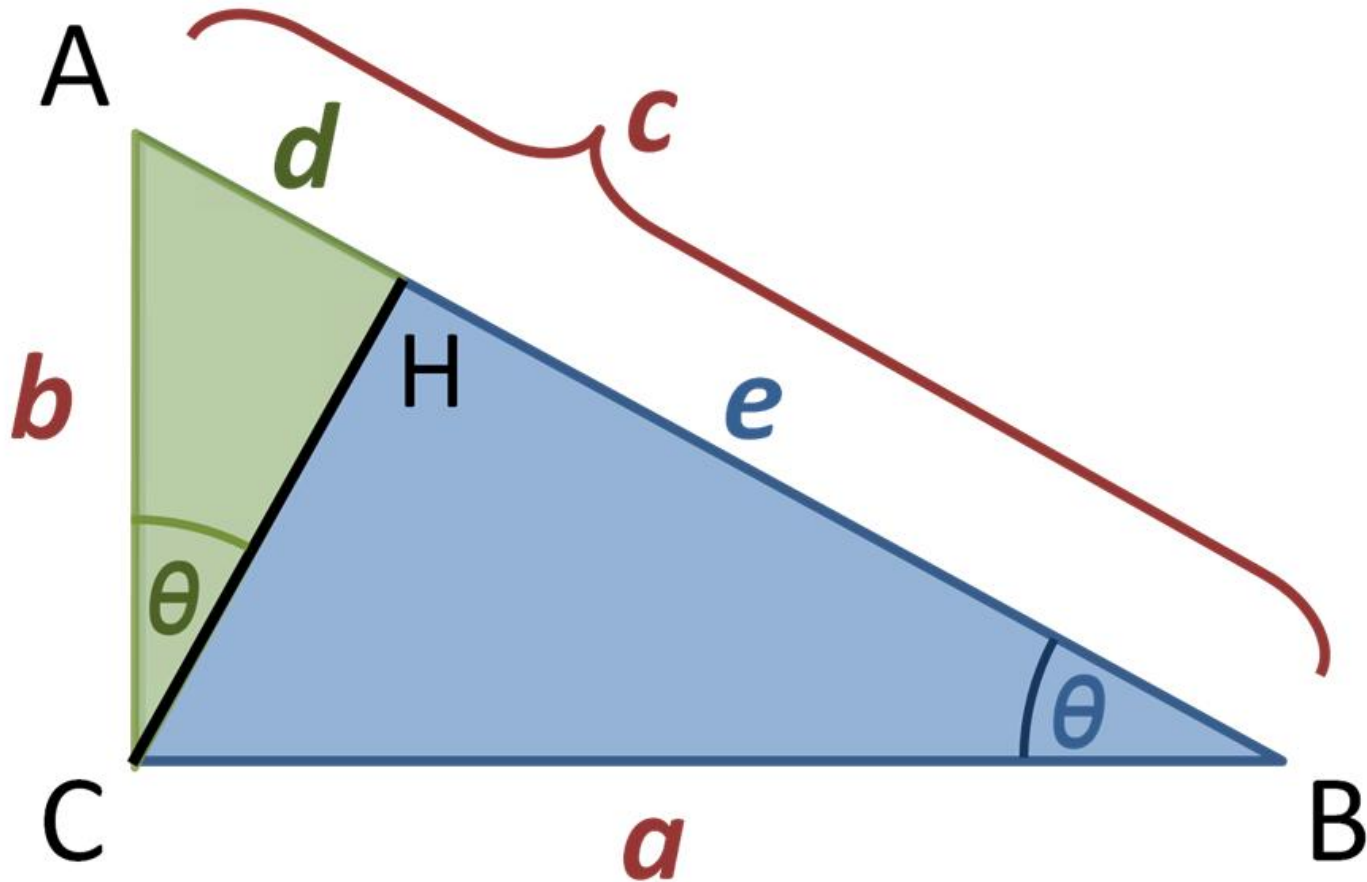


Pythagorean Formula

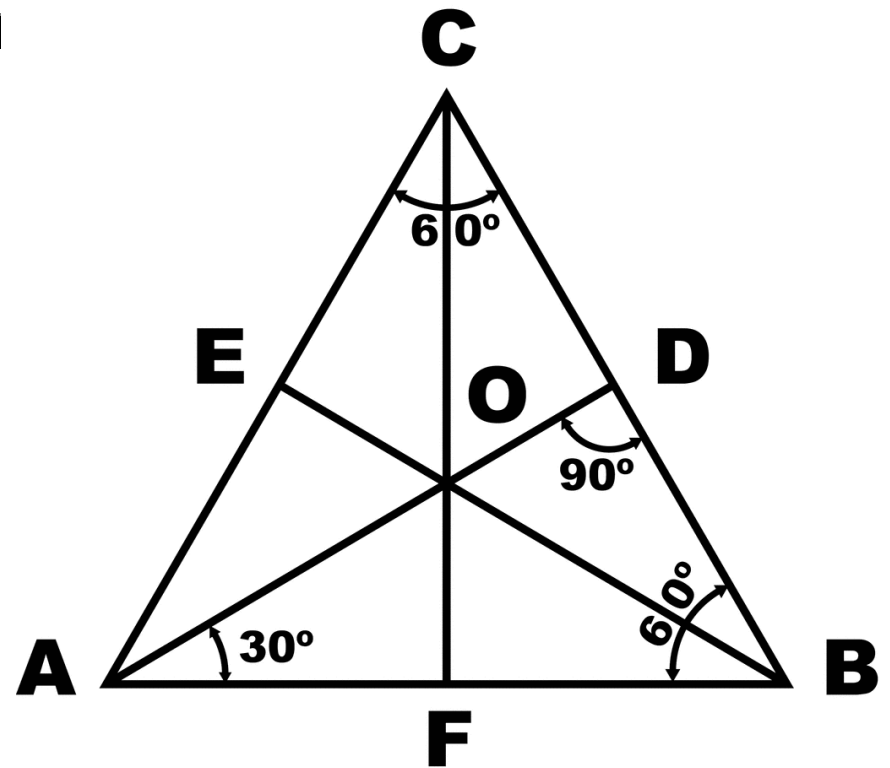
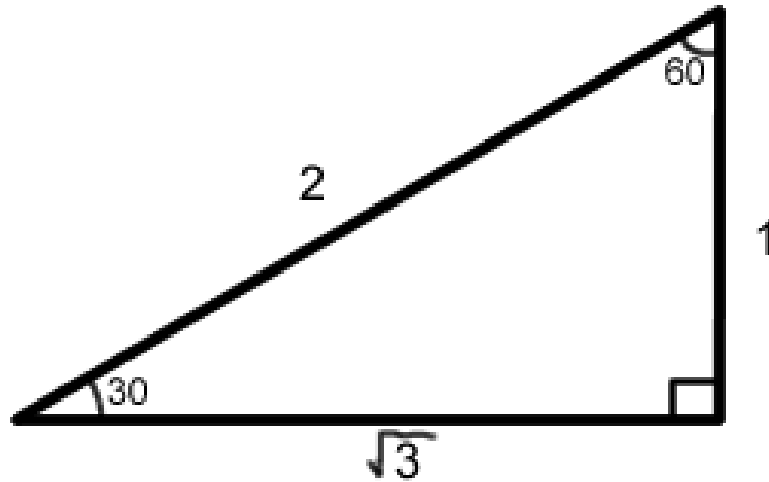


Similar Triangles

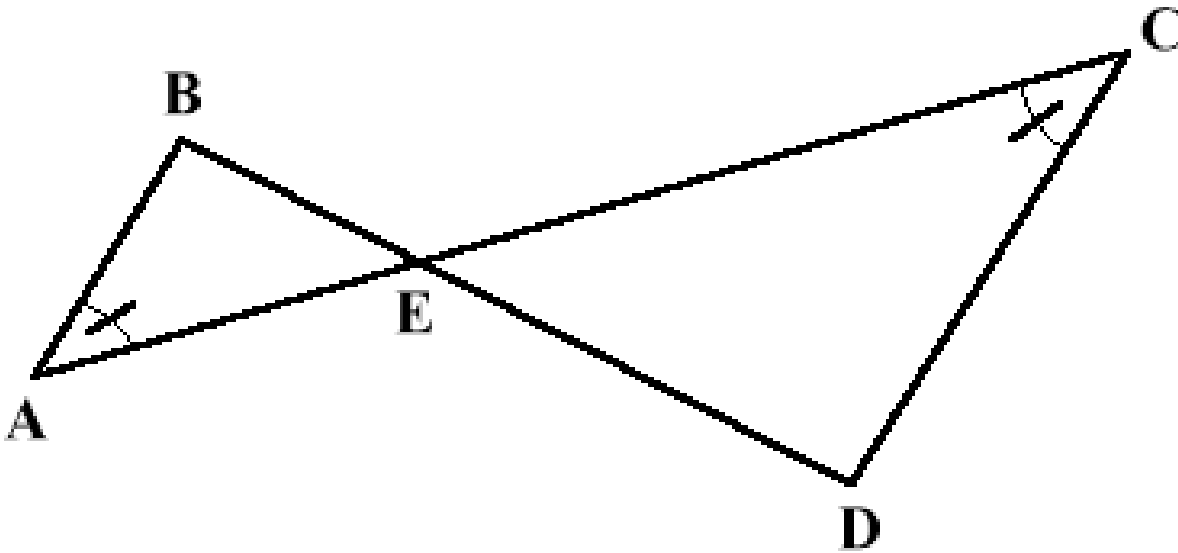
$$b/d = a/e = (d+e)/b$$



30-60-90 Triangle, Equilateral Triangle



Similar Triangles



Pythagorean Triples

- Set of three integers that satisfy the Pythagorean Theorem

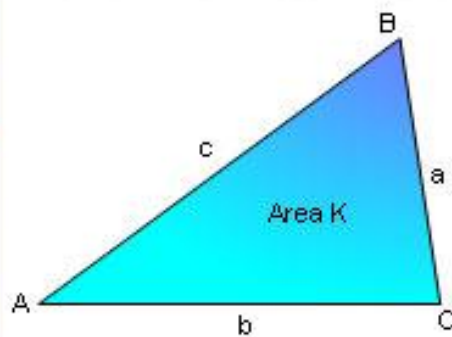
<i>m</i>	<i>n</i>	<i>a</i>	<i>b</i>	<i>c</i>
2	1	3	4	5
4	1	15	8	17
6	1	35	12	37
8	1	63	16	65
10	1	99	20	101
3	2	5	12	13
4	3	7	24	25
5	2	21	20	29
5	4	9	40	41
6	5	11	60	61



Plimpton Tablet 1900 B.C.E.

Heron's Formula

Heron's Formula



△ABC: sides a, b, c

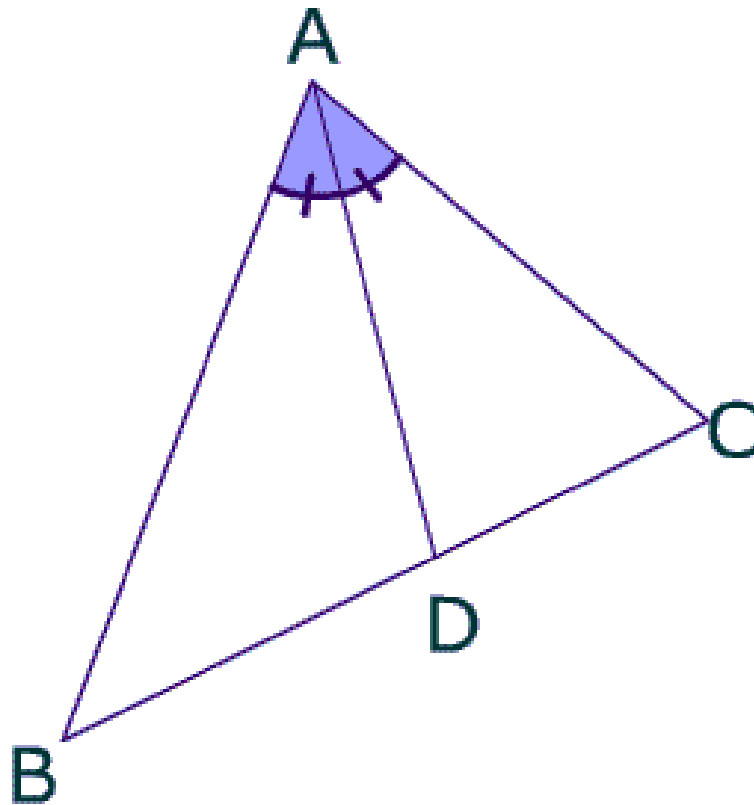
$$\text{Semiperimeter: } s = \frac{a+b+c}{2}$$

To Prove:

$$\text{Area } \triangle ABC: K = \sqrt{s(s-a)(s-b)(s-c)}$$

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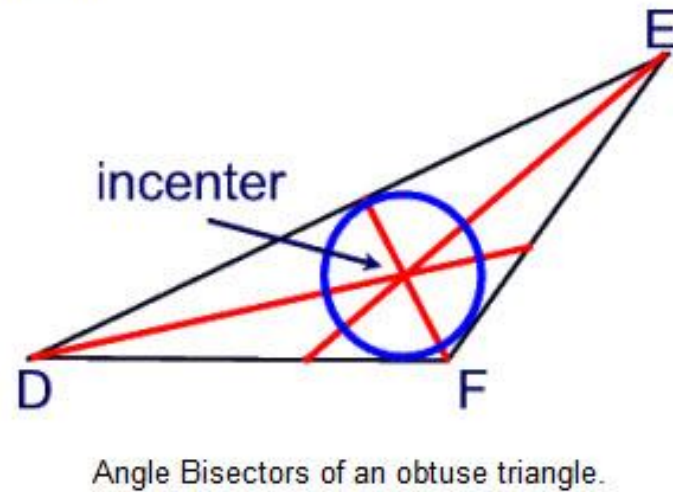
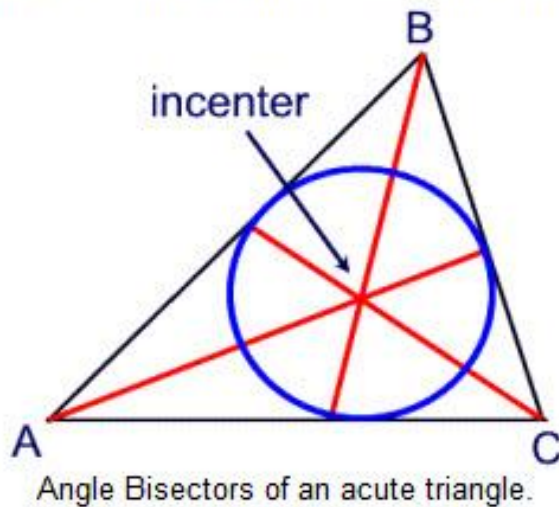
Angle Bisectors



$$\frac{\overline{CA}}{\overline{CD}} = \frac{\overline{BA}}{\overline{DB}}$$

Angle Bisectors of a triangle are concurrent!

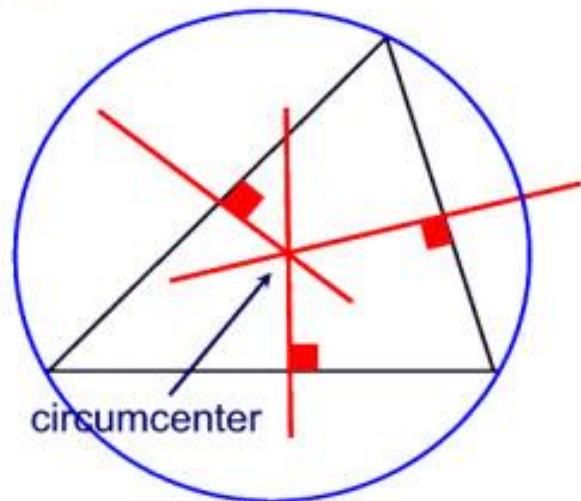
Angle Bisectors: The angle bisectors of a triangle are concurrent. Notice that the point of concurrence is in the interior of the triangles.



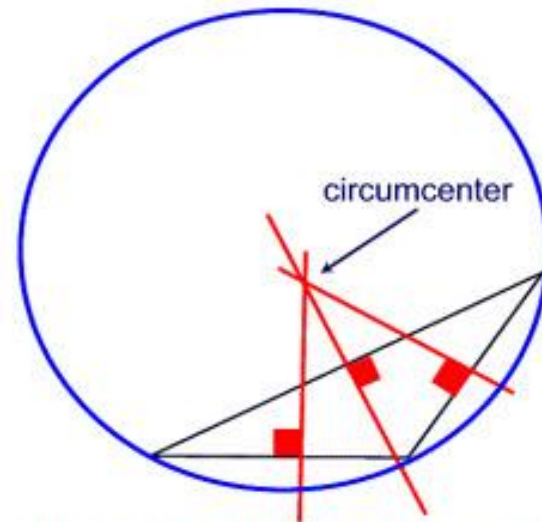
The point of concurrence is the center of an inscribed circle within the triangle. The point of concurrence is called the **incenter**.

Perpendicular Bisectors

Perpendicular Bisectors: The perpendicular bisectors of the sides of a triangle are concurrent. Notice that the point of concurrence is not necessarily in the interior of the triangles.



Perpendicular Bisectors of an acute triangle.

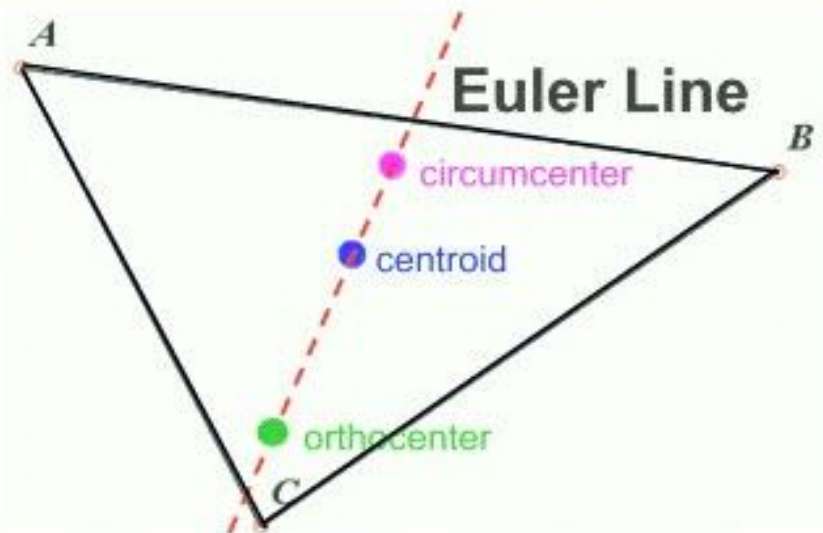


Perpendicular Bisectors of an obtuse triangle.

The point of concurrence is the center of a circumscribed circle about the triangle. The point of concurrence is called the **circumcenter**.

Euler Line

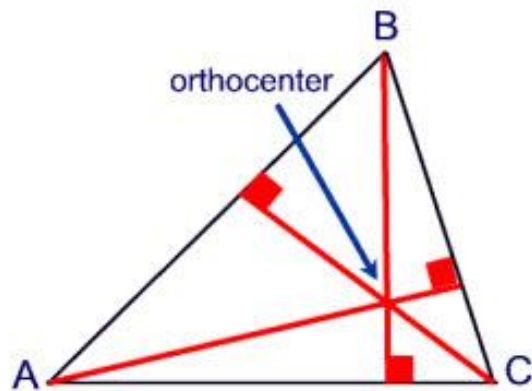
Euler Line: In any triangle, the circumcenter, centroid, and orthocenter are **collinear** (lie on the same straight line). The collinear line upon which these three points lie is called the *Euler line*. The centroid is always located between the circumcenter and the orthocenter. **The centroid is twice as close to the circumcenter as to the orthocenter.**



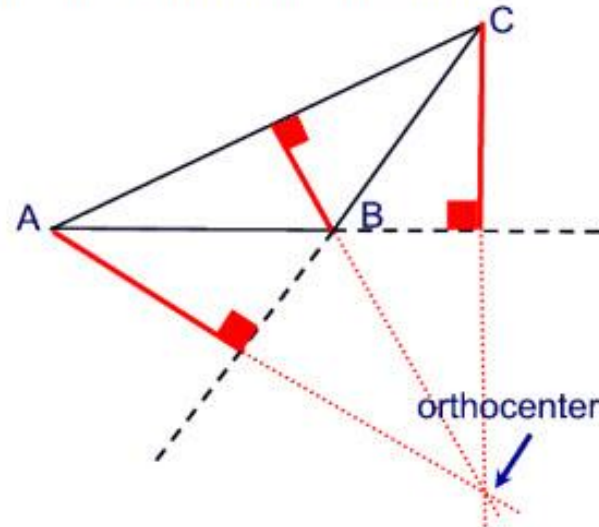
The word "Euler" is pronounced as if it were spelled "Oiler" and refers to the mathematician Leonhard Euler (1707-1783).

Altitudes

Altitudes: An altitude of a triangle is a segment from any vertex perpendicular to the line containing the opposite side. The lines containing the altitudes of a triangle are concurrent. Notice that the point of concurrence is not necessarily inside the triangle.



Altitudes of an acute triangle.

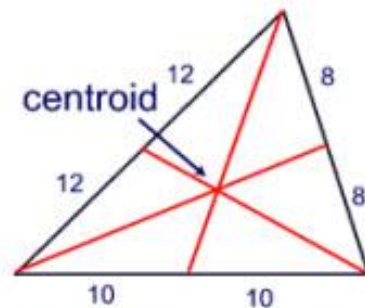


Altitudes of an obtuse triangle.

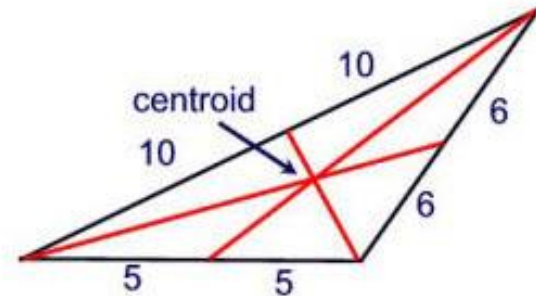
The point where the lines containing the altitudes are concurrent is called the **orthocenter** of a triangle. (The prefix "ortho" means "right".)

Medians

Medians: A median of a triangle is a segment joining any vertex to the midpoint of the opposite side. **The medians of a triangle are concurrent.** Notice that the point of concurrence is in the interior of the triangles.



Medians of an acute triangle.



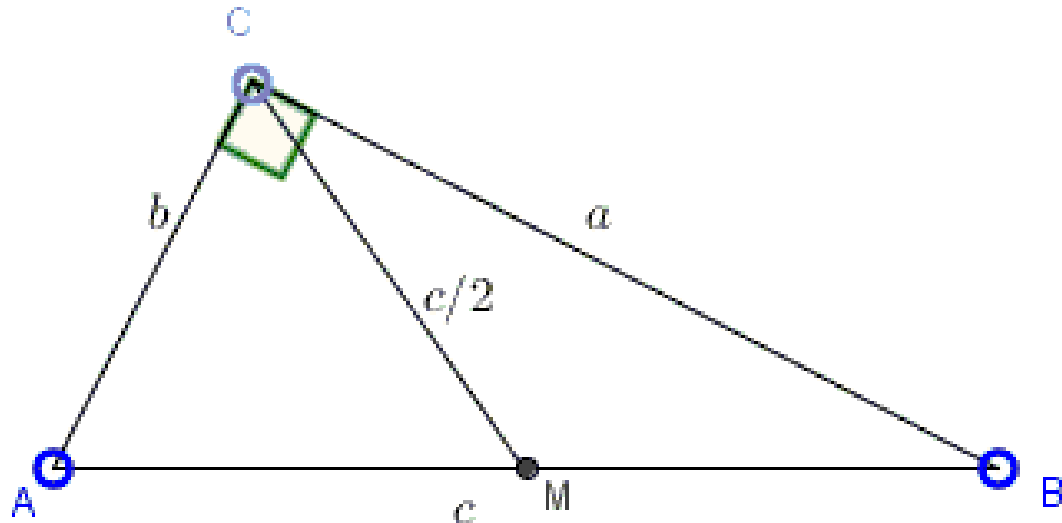
Medians of an obtuse triangle.

Archimedes showed that the point where the medians are concurrent is the center of gravity of a triangular shape of uniform thickness and density. This point where the medians are concurrent is called the **centroid** of a triangle. If you cut a triangle out of cardboard and balance it on a pointed object, such as a pencil, the pencil will mark the location of the triangle's centroid.

The centroid divides the medians into a 2:1 ratio. The section of the median nearest the vertex is twice as long as the section near the midpoint of the triangle's side.

Length of a median

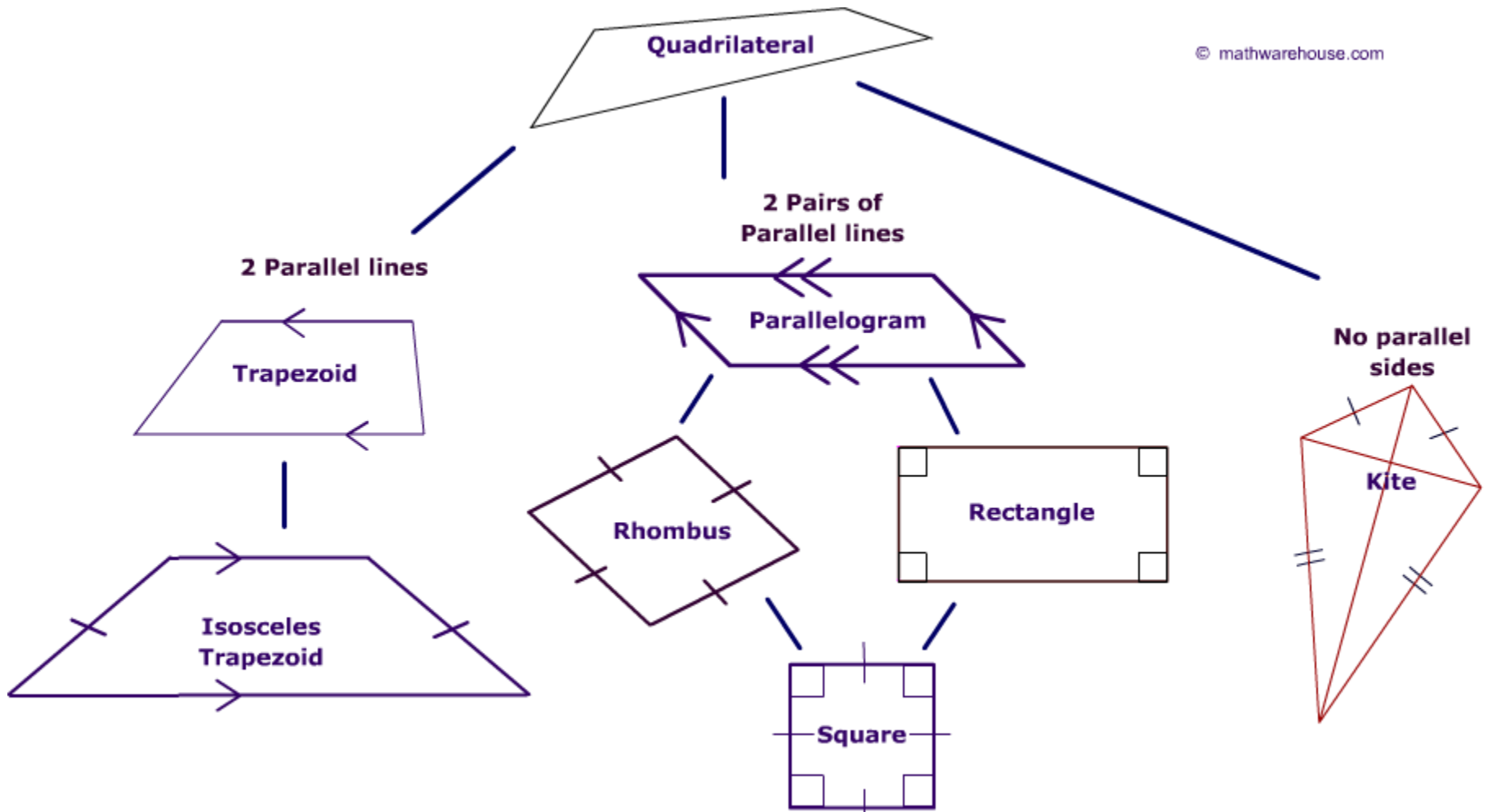
- If the length of a median is $\frac{1}{2}$ the length of the side to which it is drawn, the triangle must be a right triangle. Moreover, the side to which it is drawn is the hypotenuse



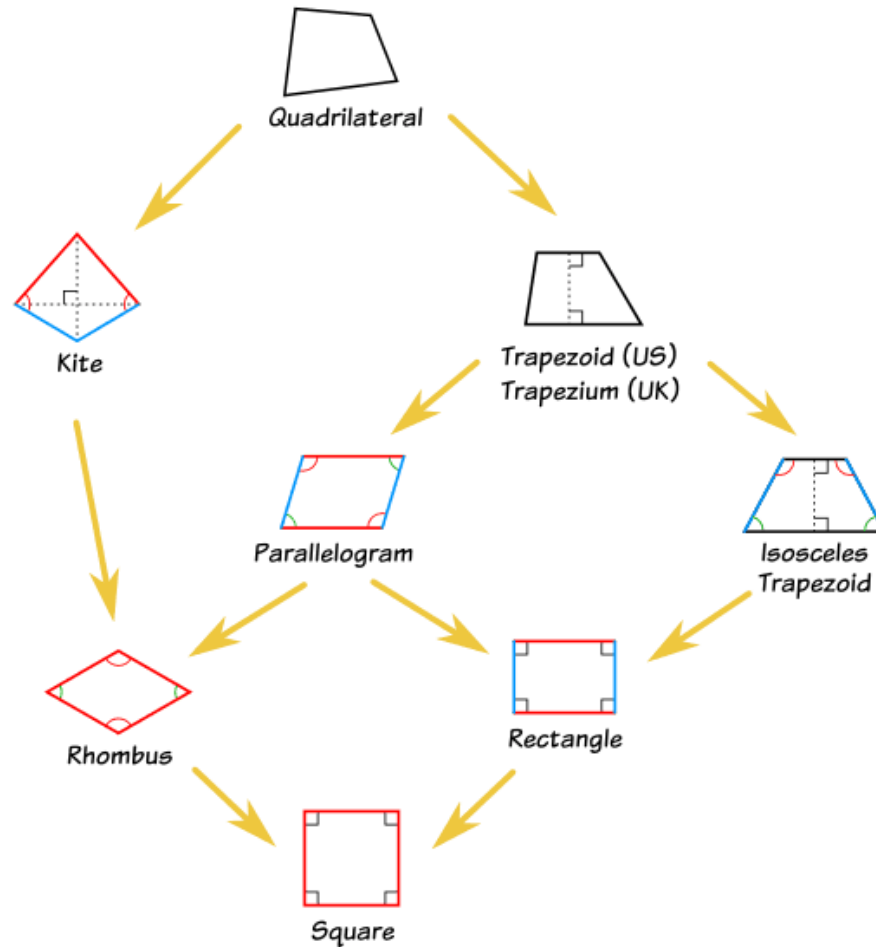
Quadrilaterals-

Interior angles = 360 degrees-

Alternative 1: Trapezoids have 1 pair of parallel sides



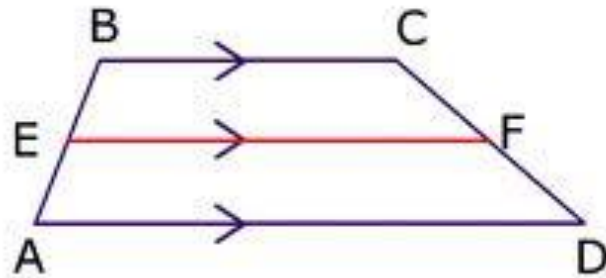
Quadrilaterals- Alternative Arrangement- Trapezoids have at least one pair of parallel sides



$$EF = \frac{1}{2}(AB + DC)$$

Trapezoids

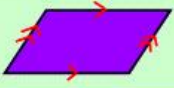
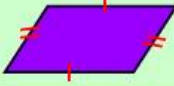
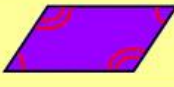



- (At least?)Two sides are parallel
- Median= Average of the bases
- Area= Height x median



$$\overline{EF} = \frac{\overline{BC} + \overline{AD}}{2}$$

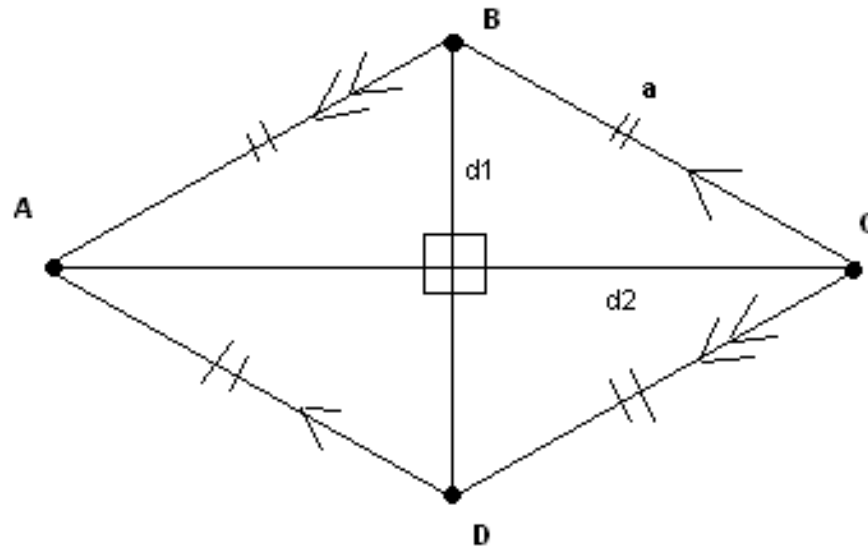
Parallelograms

- Both pairs of opposite sides equal
- Opposite Angles Equal, consecutive angles supplementary
- Diagonals Bisect each other

When GIVEN a parallelogram, the definition and theorems are stated as ...		
Sides	A parallelogram is a quadrilateral with both pairs of opposite sides parallel.	
	If a quadrilateral is a parallelogram, the 2 pairs of opposite sides are congruent. <i>(Proof appears further down the page.)</i>	
Angles	If a quadrilateral is a parallelogram, the 2 pairs of opposite angles are congruent.	
	If a quadrilateral is a parallelogram, the consecutive angles are supplementary.	
Diagonals	If a quadrilateral is a parallelogram, the diagonals bisect each other.	
	If a quadrilateral is a parallelogram, the diagonals form two congruent triangles.	

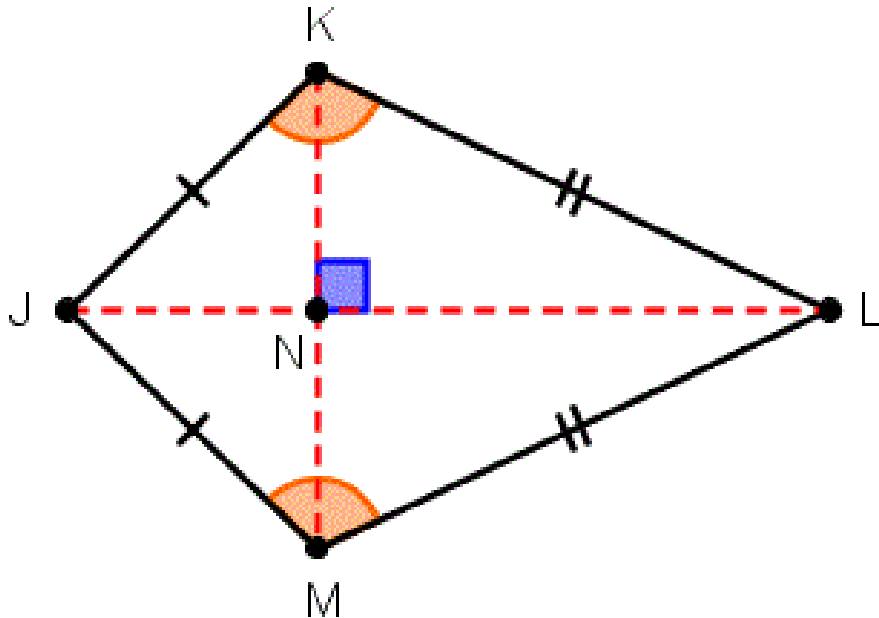
Rhombus

- All sides Equal
- Diagonals Perpendicular
- Area= half the product of diagonals



Kite

Definition: A kite is a quadrilateral with two distinct pairs of adjacent sides that are congruent.

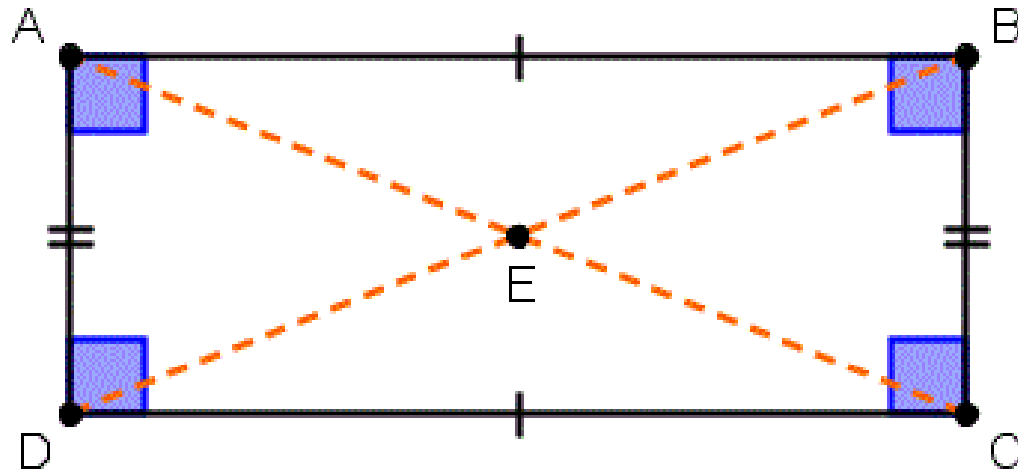


Rectangle

All angles are 90 degrees

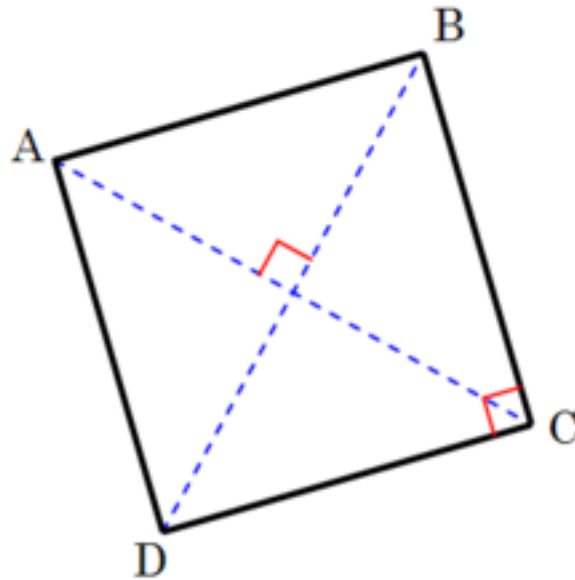
$$\text{Area} = l \times w$$

$$\text{Diagonals} = \sqrt{l^2 + w^2}$$



Square

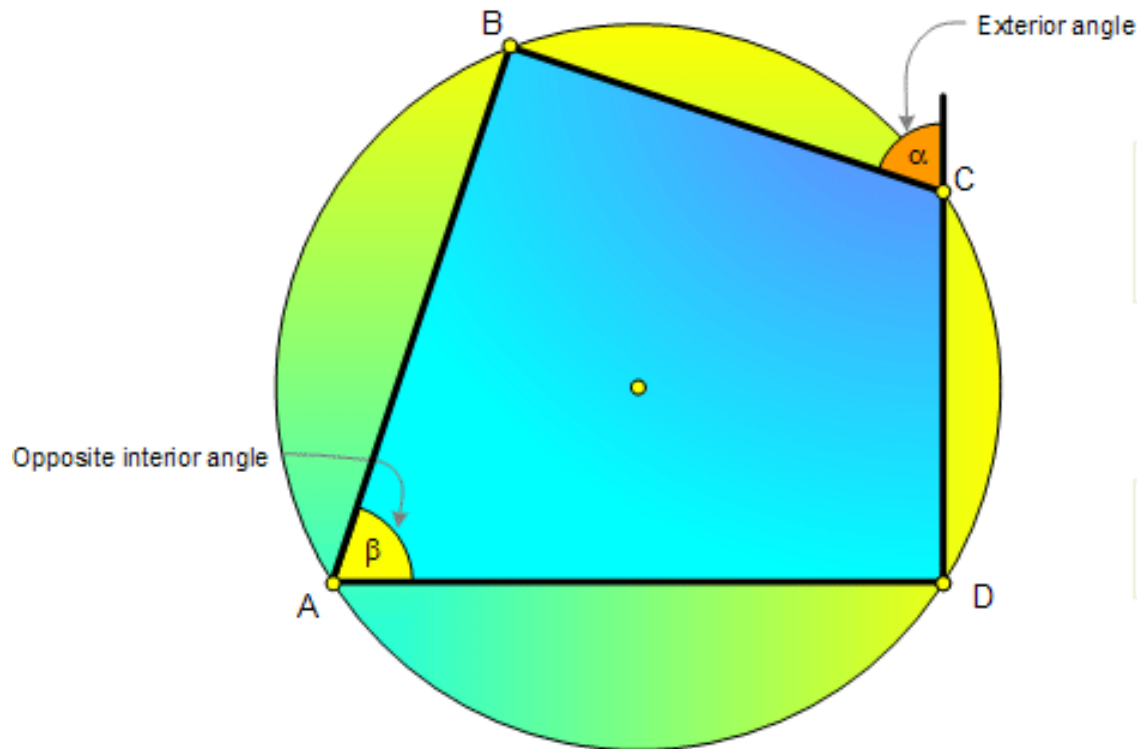
- All sides equal, all angles equal (90 degrees)
- Diagonal= side*sqrt2



Cyclic Quadrilaterals

each exterior angle is equal to the opposite interior angle.

Sum of opposite angles= 180 degrees



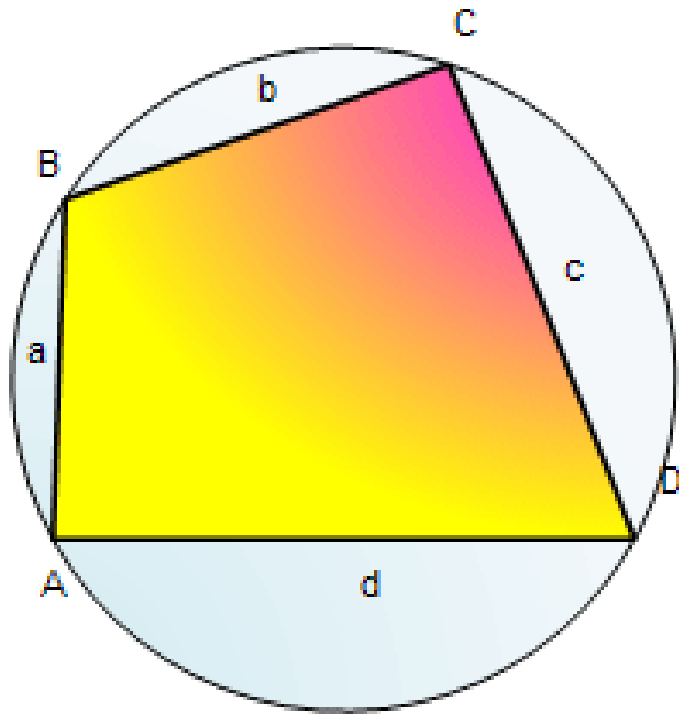
Given:
ABCD cyclic quadrilateral
 α exterior angle
 β opposite interior angle

To Prove:

$$\alpha = \beta$$

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Brahmagupta's Formula



Hypothesis

ABCD: cyclic quadrilateral

Sides: a, b, c, d

Semiperimeter $s = \frac{a+b+c+d}{2}$

To Prove:

$$\text{Area ABCD} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

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If and Only If

- Proving ‘If and Only If’ statements requires proving two different statements

“A month has less than thirty days if and only if the month is February”

To prove a statement true you must have a proof that covers all possibilities. To prove a statement false, you only have to show one counter-example.

Do not assume what you are trying to prove as part of a proof.

Related Conditionals

- If the figure is a rhombus, then it is a parallelogram; converse is false

Related Conditionals:

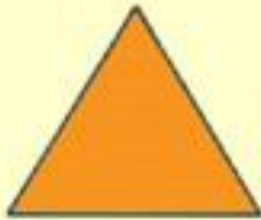
Converse: switch if and then

Inverse: negate if and then

Contrapositive: inverse of the converse
(contrapositive has the same truth value
as the original statement)

Polygons

Regular polygons



Triangle



Quadrilateral



Pentagon



Hexagon



Heptagon



Octagon



Nonagon



Decagon

Angles in a Polygon

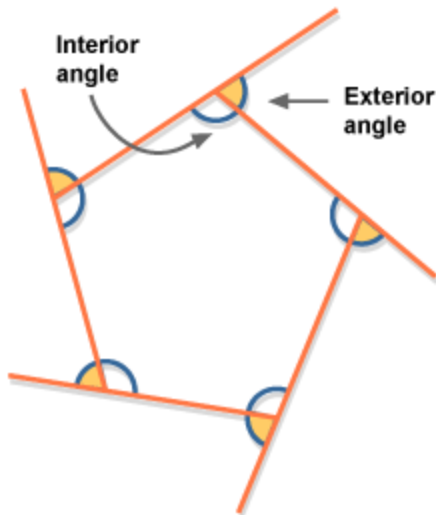
Polygon Interior/Exterior Angles:

Sum of int. angles = $180(n - 2)$

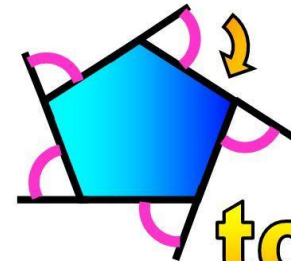
Each int. angle (regular) = $\frac{180(n - 2)}{n}$

Sum of ext. angles = 360

Each ext. angle (regular) = $\frac{360}{n}$



Exterior



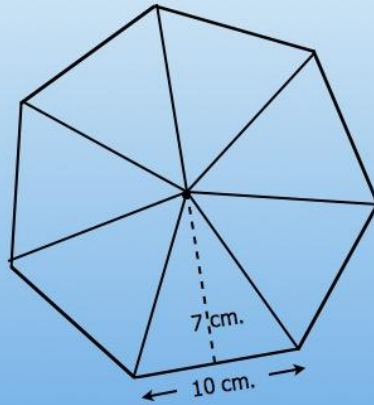
Add

to 360°

Area of a Regular Polygon

- Area = $\frac{1}{2} \times \text{perimeter} \times \text{apothem}$ (distance from center to a side)

6. Notes - Area of Regular Polygons



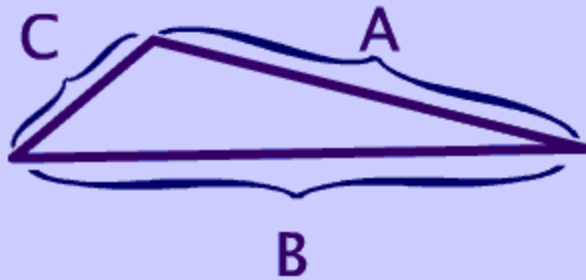
$$s = 10 \text{ cm.}$$

$$a = 7 \text{ cm.}$$

$$A = 245 \text{ square cm.}$$

Geometric Inequalities

The Triangle Inequality Theorem



$$A + B > C$$

$$B + C > A$$

$$A + C > B$$

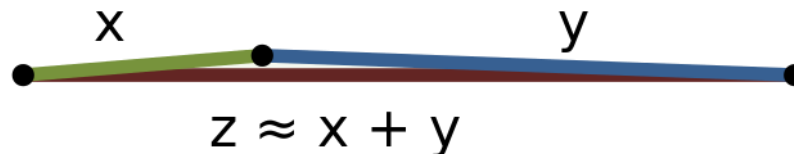
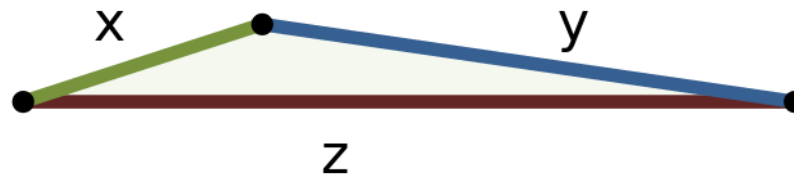
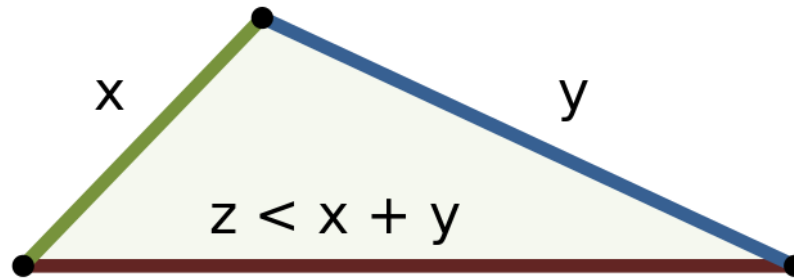
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Inequalities:

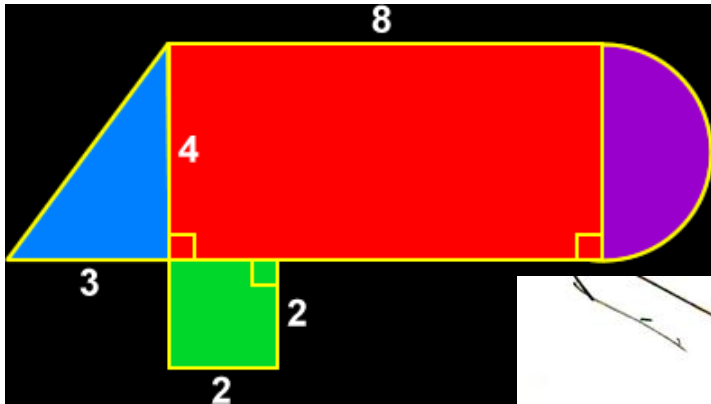
- Sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- Longest side of a triangle is opposite the largest angle.
- Exterior angle of a triangle is greater than either of the two non-adjacent interior angles.

Geometric Inequalities

- When facing problems involving lengths of altitudes of a triangle, consider using area as a tool.



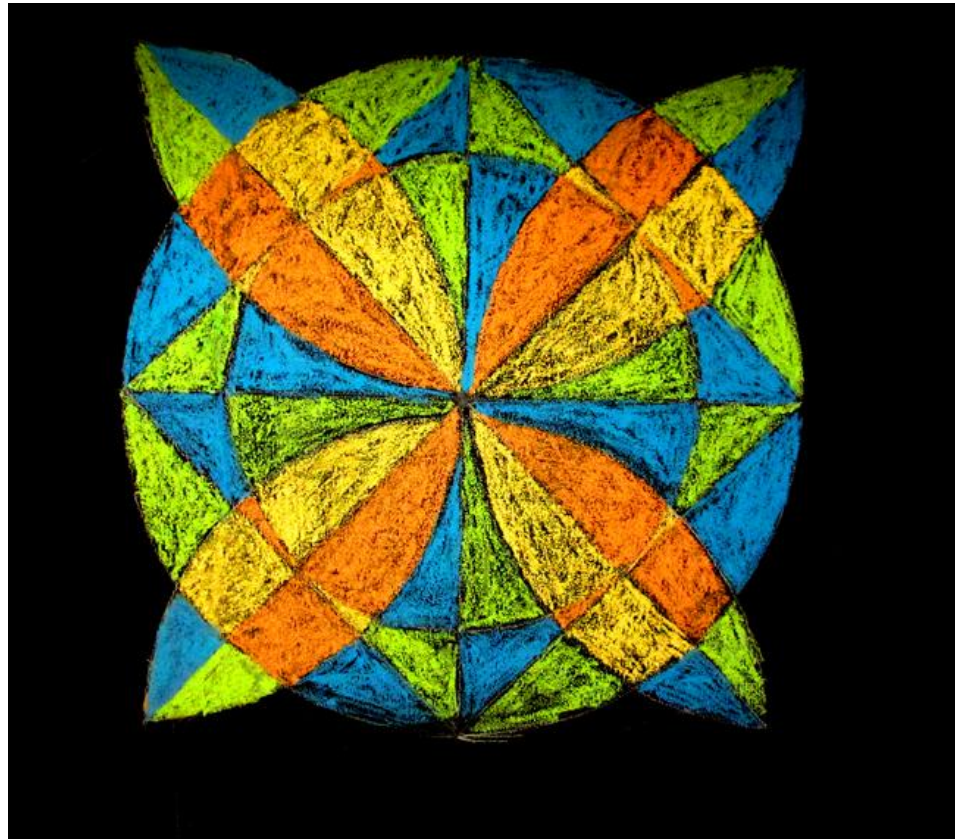
Funky Areas



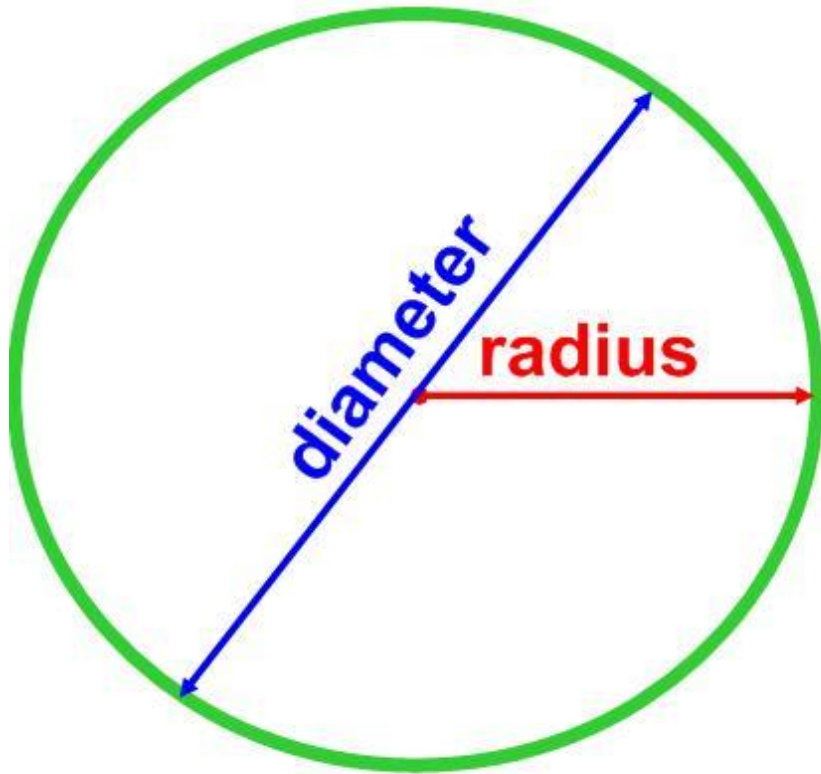
$r_3 + r_4 = 2\left(\arccos\frac{2}{3}(36\pi) - 6\sqrt{5}\right)$
 $r_2 + r_1 = 36\pi - 2\left(\arccos\frac{2}{3}(36\pi) - 6\sqrt{5}\right)$

$r_2 + r_3 = ?$
 $r_2 + r_4 = 12\pi - 9\sqrt{5}$
 $r_1 + r_3 = 24\pi + 9\sqrt{5}$
 $r_3 + r_4 = 2\left(\arccos\frac{2}{3}(36\pi) - 6\sqrt{5}\right)$
 $r_2 + r_1 = 36\pi - 2\left(\arccos\frac{2}{3}(36\pi) - 6\sqrt{5}\right)$

Funky Areas



Circles



Area of a circle
 $= \pi \times \text{radius}^2$

Circumference of a
circle $= \pi \times \text{diameter}$

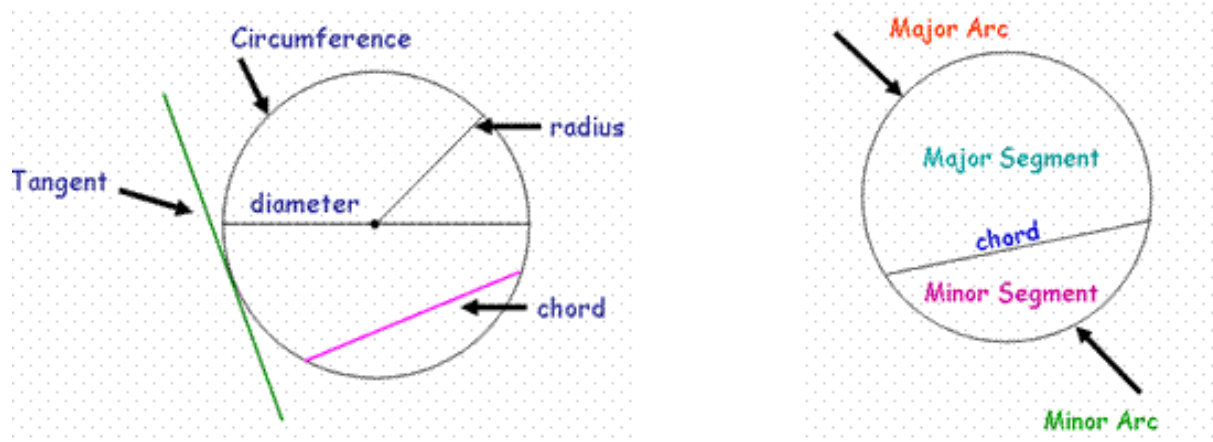
remember that the
 $\text{diameter} = 2 \times \text{radius}$

Circles

3. Circle Theorems

Parts of a Circle...

Before we start going through each of the circle theorems, it is important we know the names for each part of the circle, as we will be using these terms in this section.



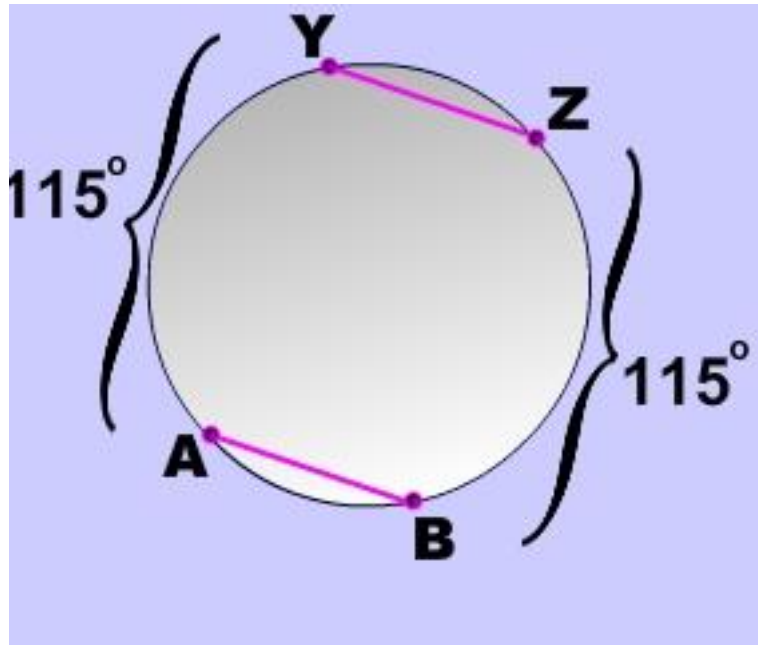
Three things you should Learn about Circle Theorems:

- 1) What each of the theorems say
- 2) How to spot them
- 3) How to show you are using circle theorems in your answers

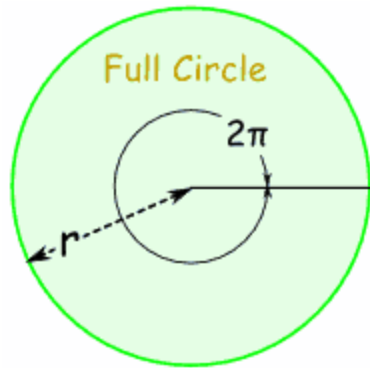
And if you can do all these, then that's a pretty tricky topic all sorted!

Chords of a Circle

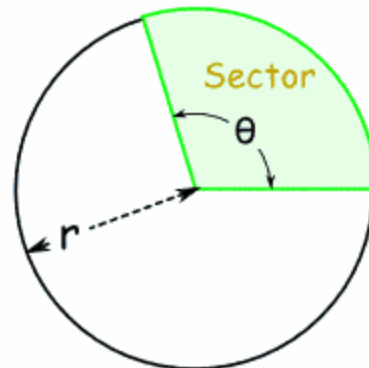
If chords of a circle are equal in length,
they subtend equal arcs



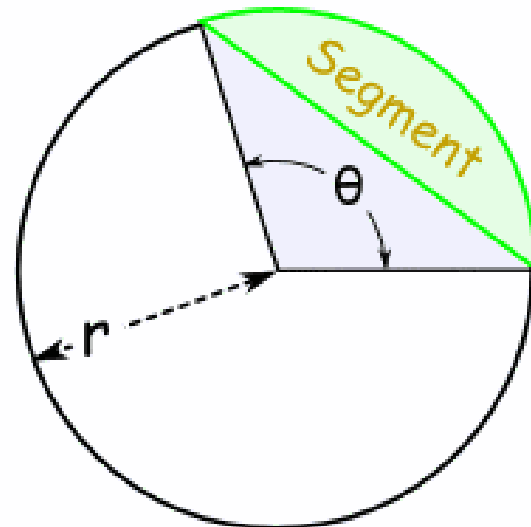
Area of a Sector



$$A = \pi \times r^2$$



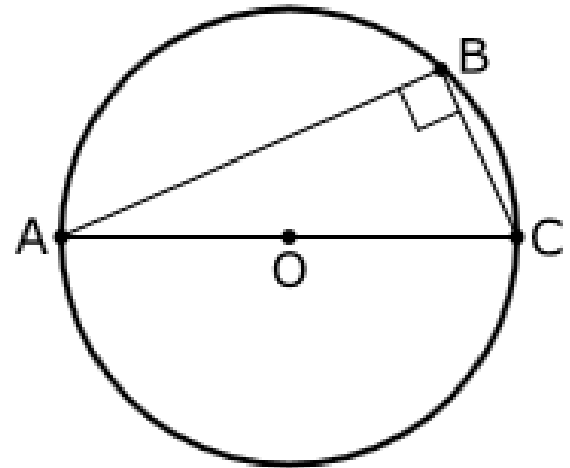
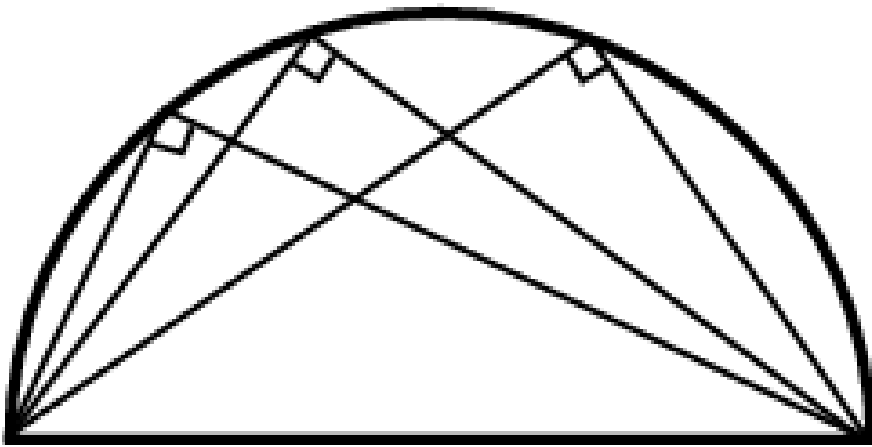
$$A = \left(\frac{\theta}{2\pi}\right) \times \pi \times r^2$$
$$= \left(\frac{\theta}{2}\right) \times r^2$$



$$A = \frac{1}{2} \times (\theta - \sin \theta) \times r^2$$

Thale's Theorem

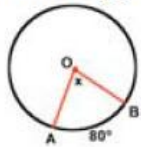
- Any Angle Inscribed in a semicircle is a right angle (Thale's Theorem)



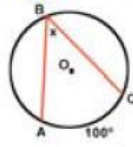
Circle Angles

Circle Angles:

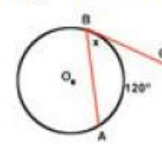
Central angle = arc



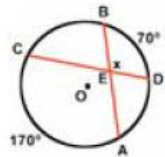
Inscribed angle = half arc



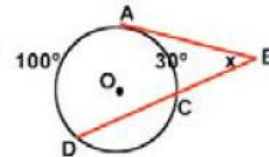
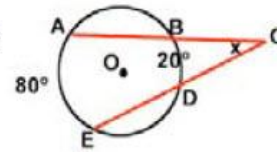
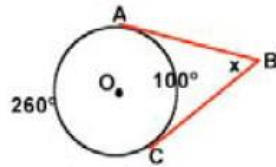
Angle by tangent/chord = half arc



Angle formed by 2 chords
= half the sum of arcs

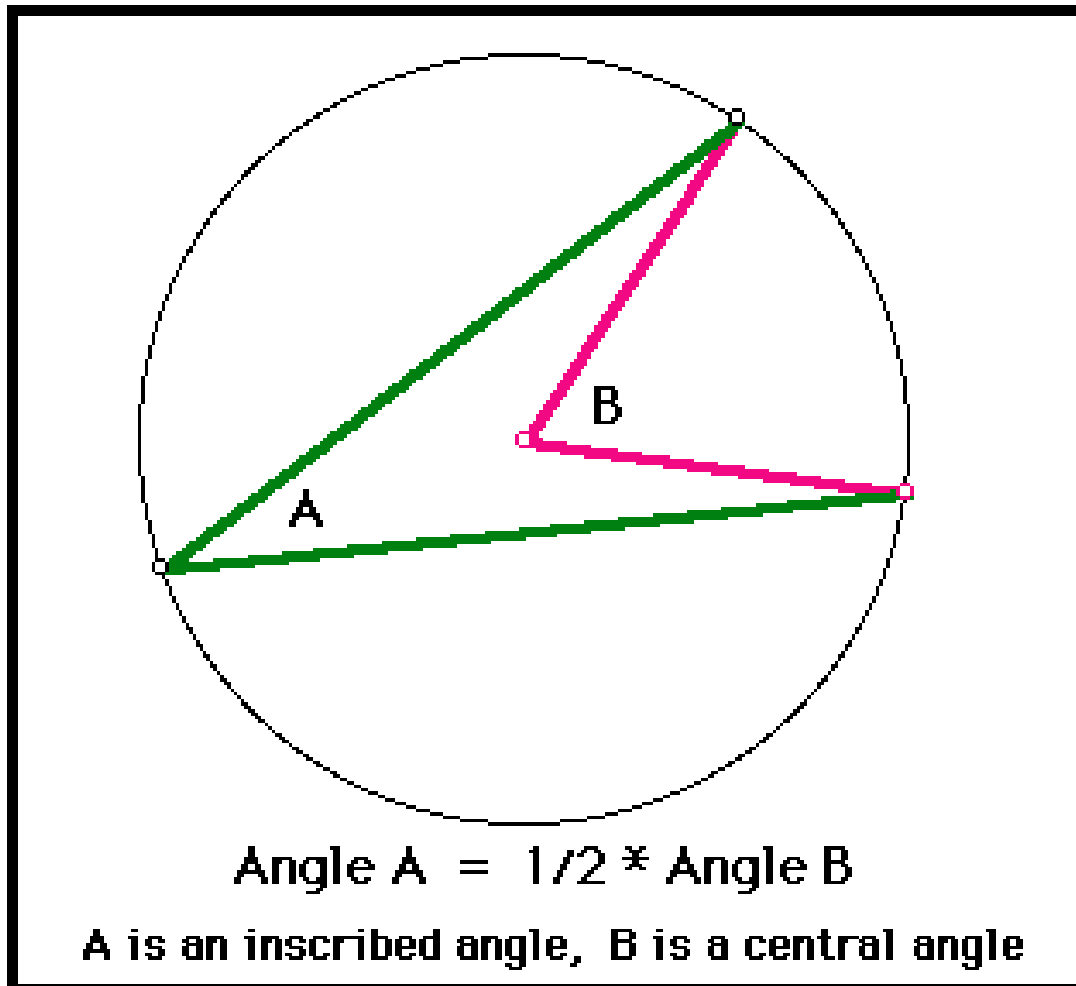


Angle formed by 2 tangents, or 2 secants, or a tangent/secant
= half the difference of arcs



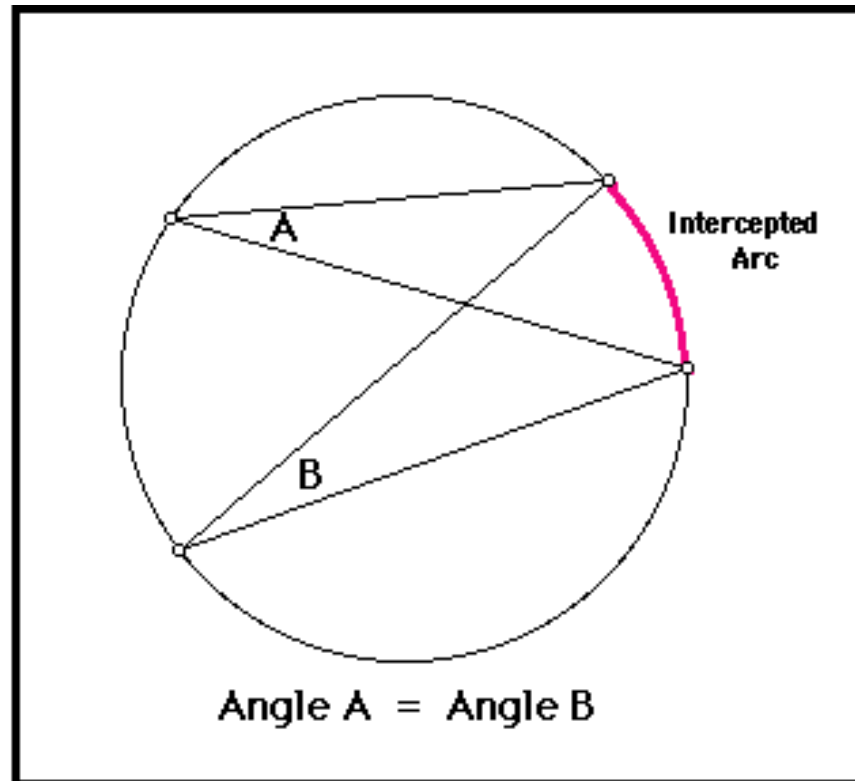
Inscribed Angles

The measure of an inscribed angle is $\frac{1}{2}$ the arc it intercepts



Inscribed Angles

Any two angles inscribed in the same arc are equal

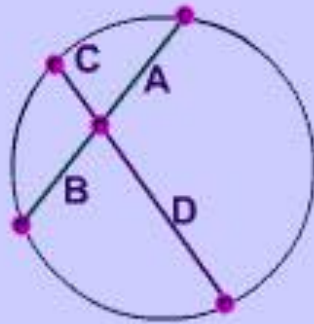


Intersecting Chords

Theorems involving the chord of a circle

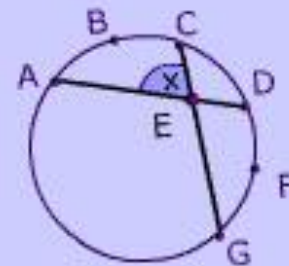
Product of segments theorem

$$A \cdot B = C \cdot D$$



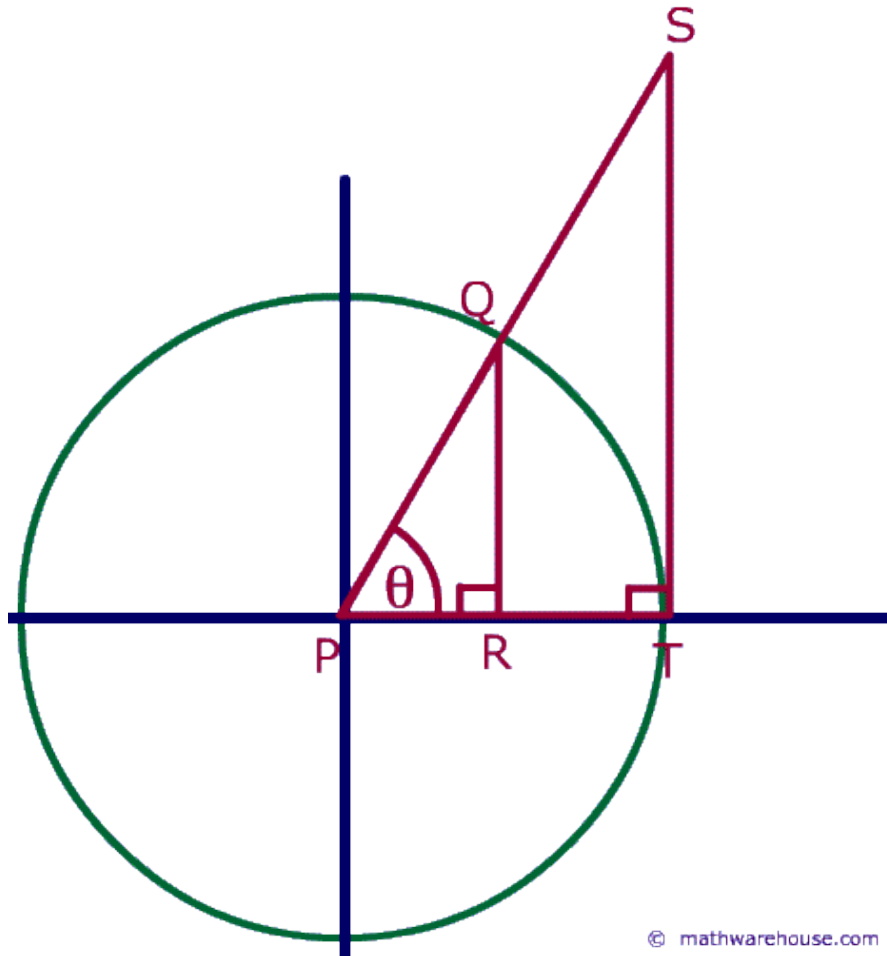
Intersecting Chords Angles & Arcs of Intersecting Chords

$$\angle X = \frac{1}{2}(\widehat{ABC} + \widehat{DFG})$$

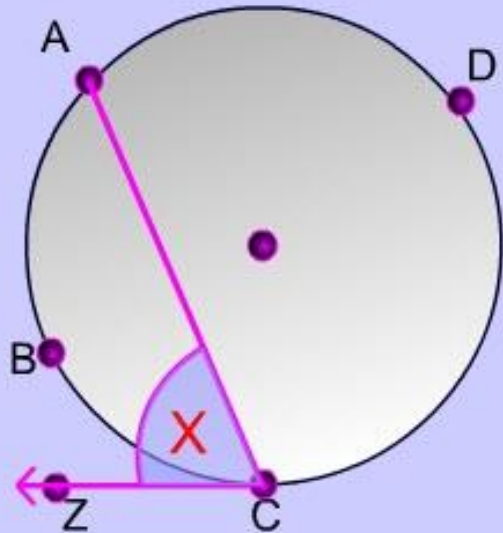


Tangents

A Tangent Line is perpendicular to a radius



Tangent Chords



The Theorem: An Angle formed by a **chord** and a **tangent** that intersect on a **circle** is half the measure of the **intercepted arc**

Look at the picture on the left

$$x = \frac{1}{2} m \widehat{ABC}$$

This means that the measure of **arc** ABC (the purple portion of the circle itself) is twice the measure of **angle** C.

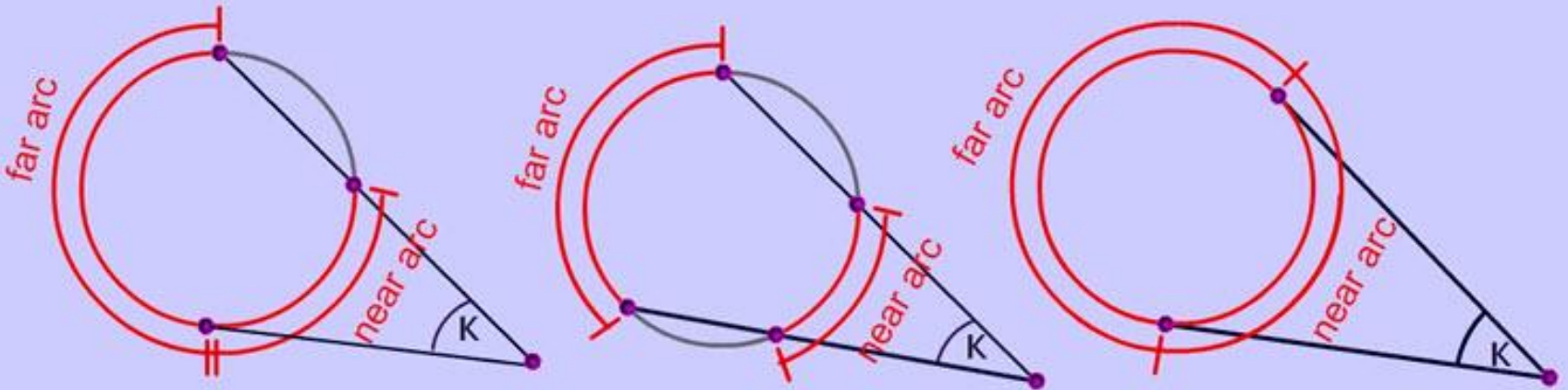
$$x = \frac{1}{2} m \widehat{ABC}$$

Secant-Secant Chords

Far Arc – Near Arc Formula

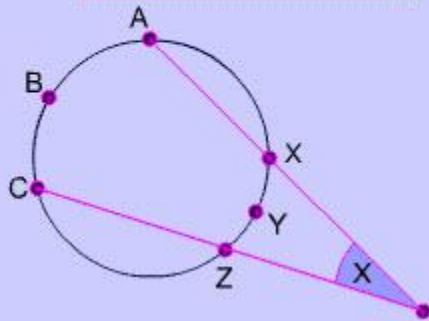
All of the formulas on this page can be thought of in terms of a "far arc" and a "near arc". The angle formed outside of the circle is always equal to the the far arc minus the near arc divided by 2.

$$m\angle K = \frac{(\text{far arc} - \text{near arc})}{2}$$

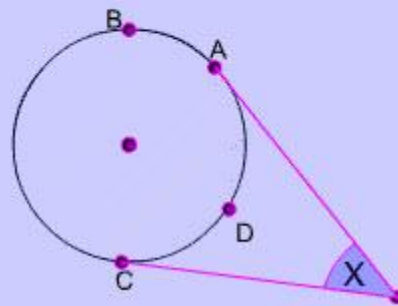


Secant-Secant Chords

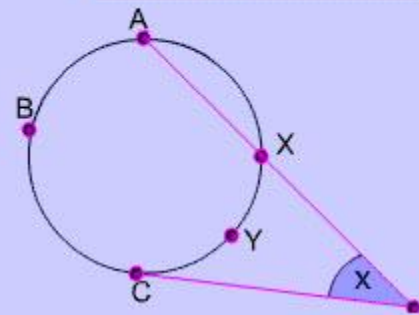
I $m\angle X = \frac{1}{2}(\widehat{ABC} - \widehat{XYZ})$



III $m\angle X = \frac{1}{2}(\widehat{ABC} - \widehat{CDA})$

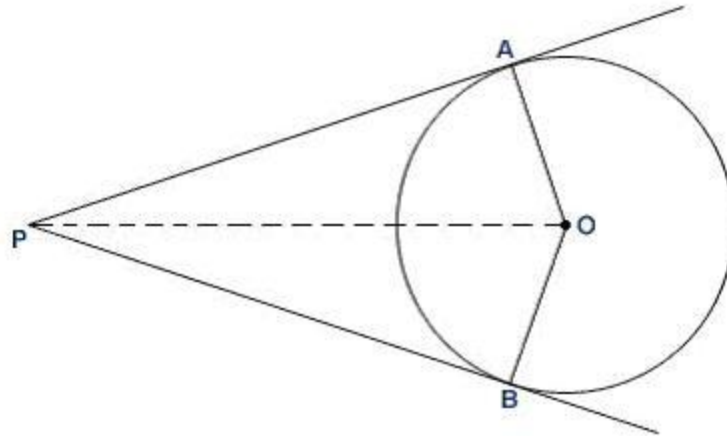


II $m\angle X = \frac{1}{2}(\widehat{ABC} - \widehat{XYC})$



Two tangents are equal!

- Hat Rule:



Power of a Point Theorem:

Suppose a line through a point P intersects a circle in two points, U and V. For all such lines, the product $PU \cdot PV$ is a constant.

Circle Segments

In a circle, a radius perpendicular to a chord bisects the chord.

Intersecting Chords Rule:

$$(\text{segment part}) \cdot (\text{segment part}) = (\text{segment part}) \cdot (\text{segment part})$$

Secant-Secant Rule:

$$(\text{whole secant}) \cdot (\text{external part}) = (\text{whole secant}) \cdot (\text{external part})$$

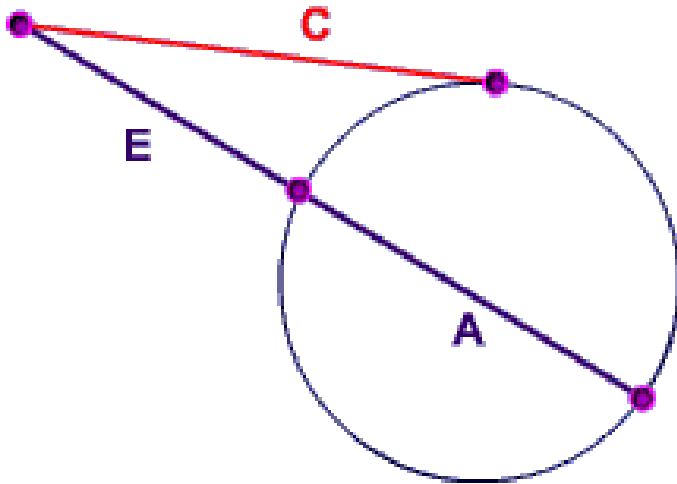
Secant-Tangent Rule:

$$(\text{whole secant}) \cdot (\text{external part}) = (\text{tangent})^2$$

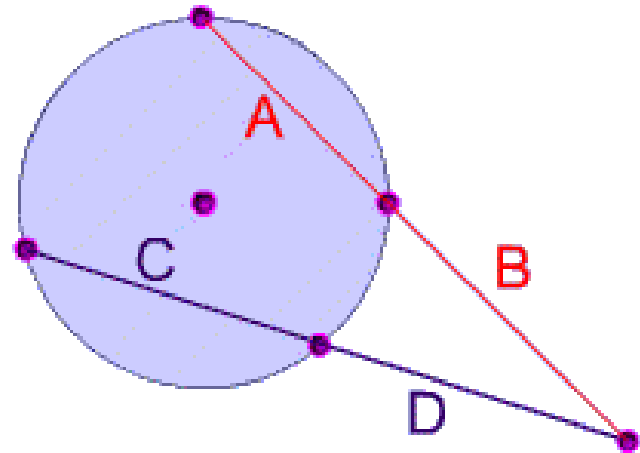
Hat Rule: Two tangents are equal.

Power of a Point

$$C^2 = E \cdot (A + E)$$

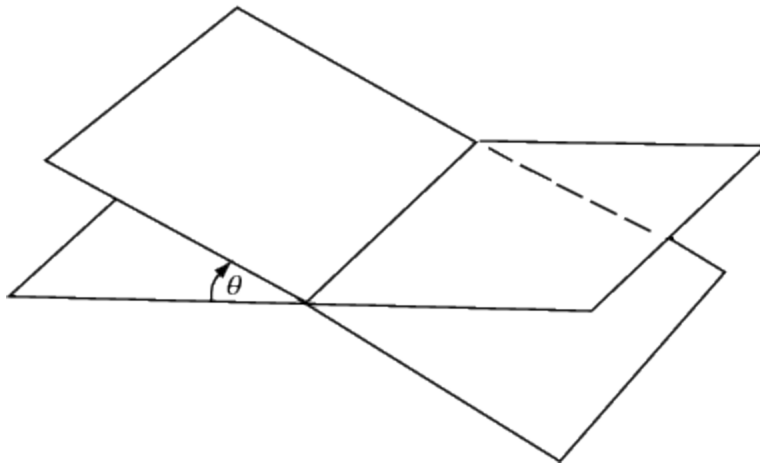


$$(A+B) \cdot B = (C+D) \cdot D$$

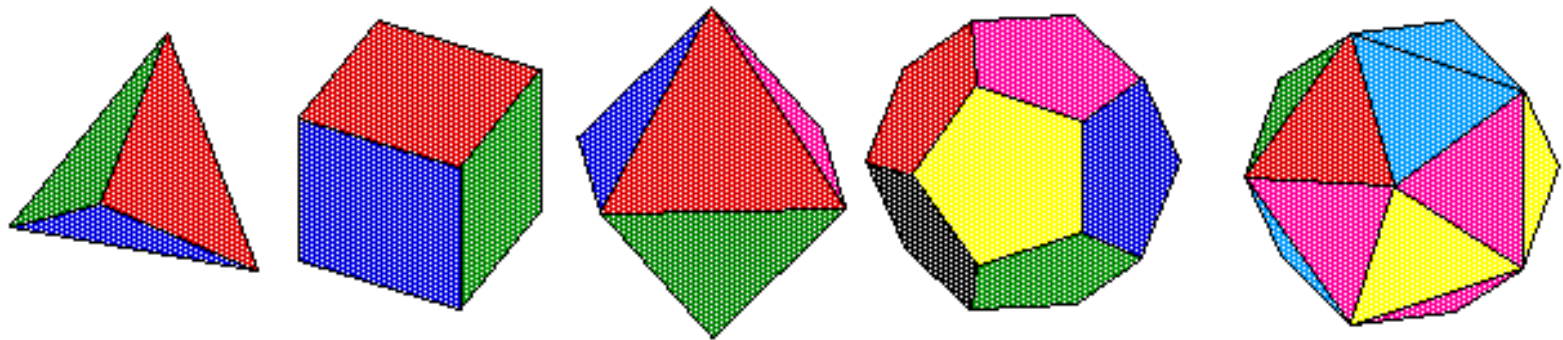


Planes

- Three non-collinear points determine a plane
- Dihedral Angle: angle between the planes



The five Platonic solids



The Tetrahedron

The Cube

The Octahedron

The Dodecahedron

The Icosahedron

The five regular solids discovered by the Ancient Greek mathematicians are:

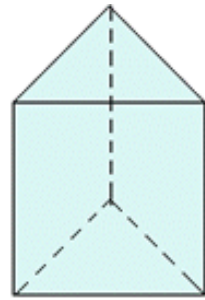
The Tetrahedron :	4 vertices	6 edges	4 faces	each with 3 sides
The Cube :	8 vertices	12 edges	6 faces	each with 4 sides
The Octahedron :	6 vertices	12 edges	8 faces	each with 3 sides
The Dodecahedron :	20 vertices	30 edges	12 faces	each with 5 sides
The Icosahedron :	12 vertices	30 edges	20 faces	each with 3 sides

The solids are regular because the same number of sides meet at the same angles at each vertex and identical polygons meet at the same angles at each edge.

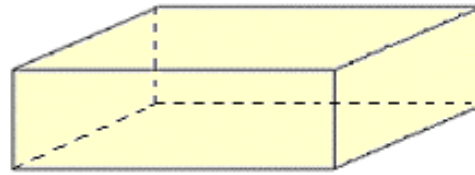
These five are the only possible regular polyhedra.

Prisms

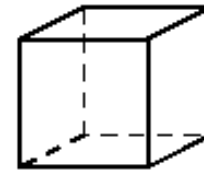
Volume = base x height



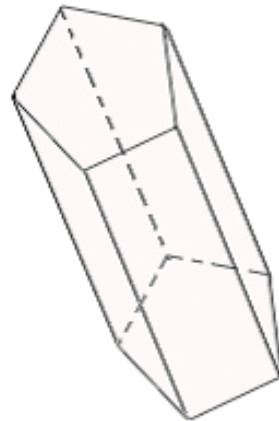
Triangular Prism



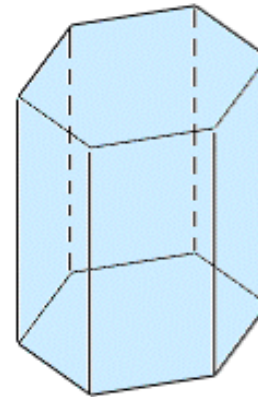
Rectangular Prism



Cube



Pentagonal Prism



Hexagonal Prism

3-D Figures

3-D Figures:

Prism: $V = Bh$

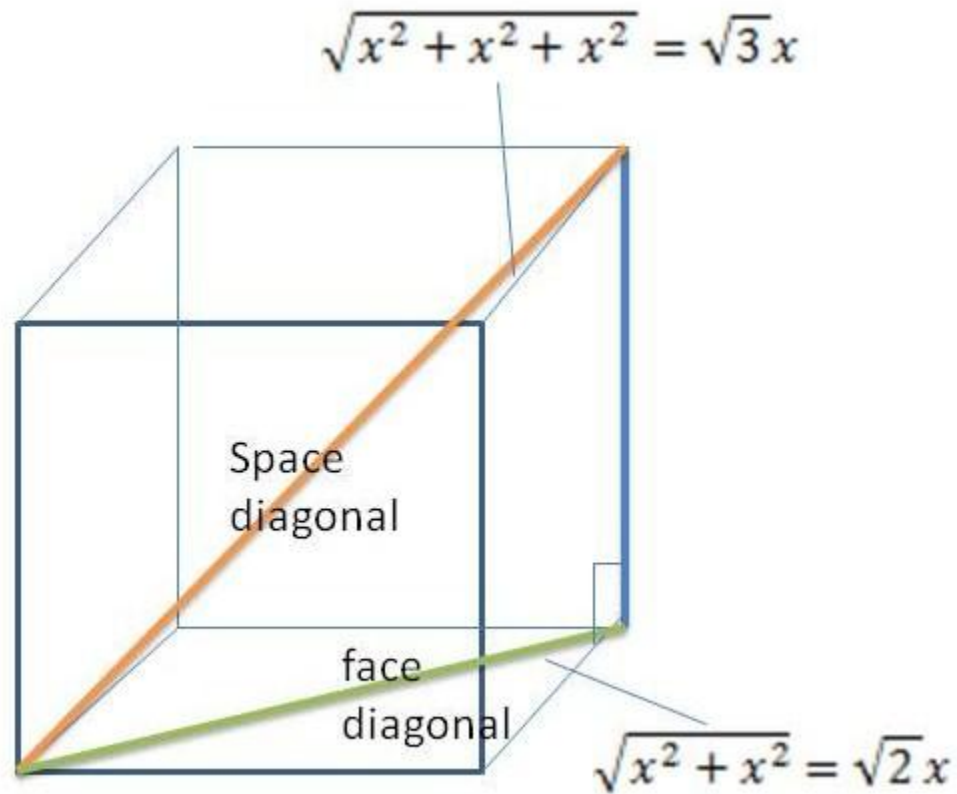
Pyramid: $V = \frac{1}{3}Bh$

Cylinder: $V = \pi r^2 h$; $SA = 2\pi rh + 2\pi r^2$

Cone: $V = \frac{1}{3}\pi r^2 h$; $SA = s\pi r + \pi r^2$

Sphere: $V = \frac{4}{3}\pi r^3$; $SA = 4\pi r^2 = \pi d^2$

Cube



Pyramid

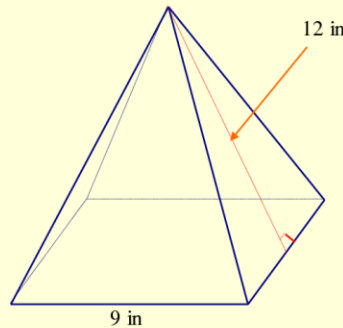
Volume = $\frac{1}{3}$ * area of base * height

Lateral Surface Area = $\frac{1}{2}$ Perimeter * slant height

Lateral and Surface Area of a Pyramid

$$LA = \frac{Pl}{2}$$

P = perimeter of the base
l = the slant height



$$SA = LA + B$$

B = area of the base

Cylinders

Cylinder

Surface Area

We will need to calculate the surface area of the top, base and sides.

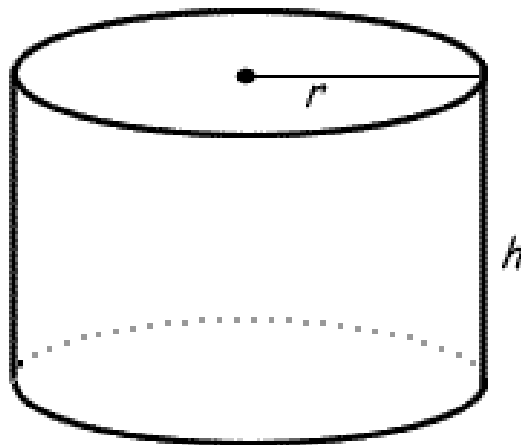
Area of the top is πr^2

Area of the bottom is πr^2

Area of the side is $2\pi rh$

Therefore the
Formula is:

$$A = 2\pi r^2 + 2\pi rh$$



Volume

$$V = \pi r^2 h$$

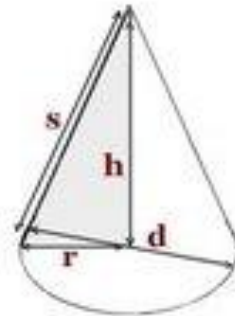
Cones

$$\text{Volume} = \frac{1}{3} * \pi * r^2 * h$$

$$\text{LSA} = \pi * r * s$$



Cone



$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

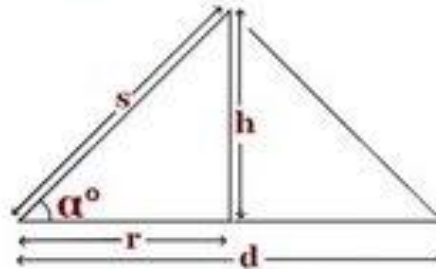
$$\text{Surface} = \pi r s$$

s = Slant height

h = Height

d = Diameter

r = Radius



$$\tan \alpha^\circ = h/r$$

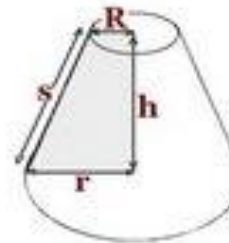
$$\sin \alpha^\circ = h/s$$

$$\cos \alpha^\circ = r/s$$

$$s^2 = h^2 + r^2$$



**Frustum
of a cone**



$$\text{Volume} = \frac{1}{3} \pi h (R^2 + r^2 + Rr)$$

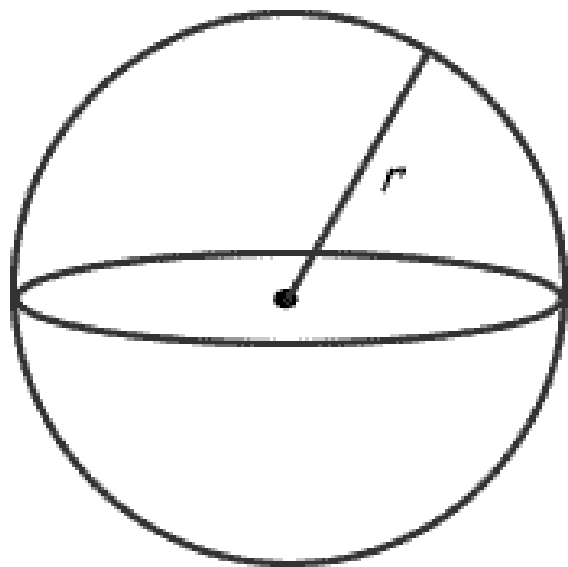
$$\text{Surface} = \pi s (R + r)$$

Spheres

Sphere

Surface
Area

$$A = 4\pi r^2$$



Volume

$$V = \frac{4}{3}\pi r^3$$

Transformations

Transformations

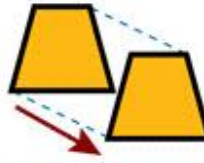
A transformation is the movement of a figure in a given plane



Reflection
Flip over a given line.



Rotation
Turn around a central point.



Translation
Move or slide with no rotation or reflection.

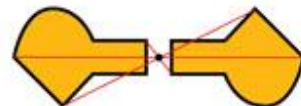
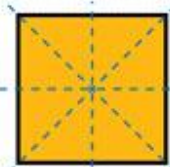
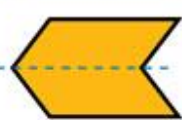
Symmetry

Symmetry is when one shape is exactly like another if you fold it over a line or turn it.



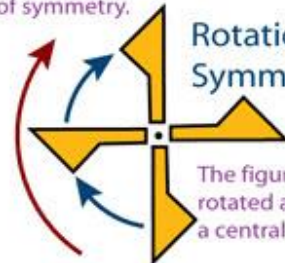
Reflection Symmetry

The figures are mirror images when folded over a line of symmetry. Figures can have more than one line of symmetry.



Point Symmetry

The figure has matching parts the same distance from a central point but in the opposite direction.

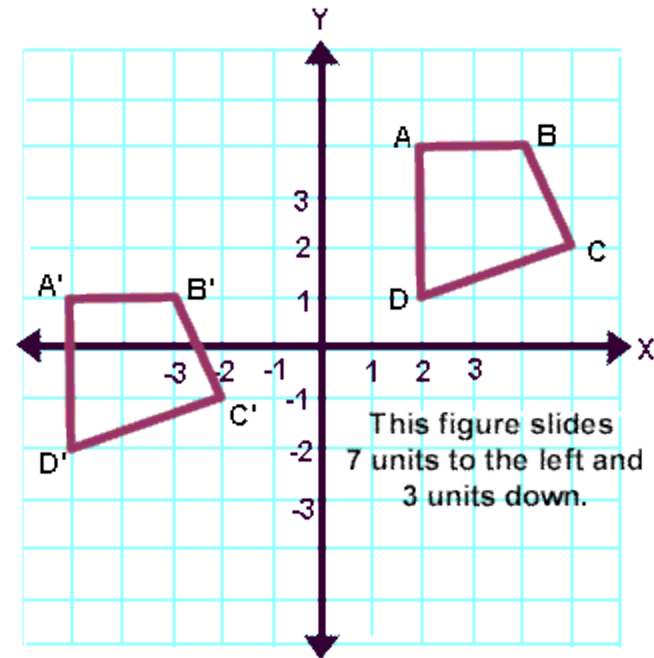
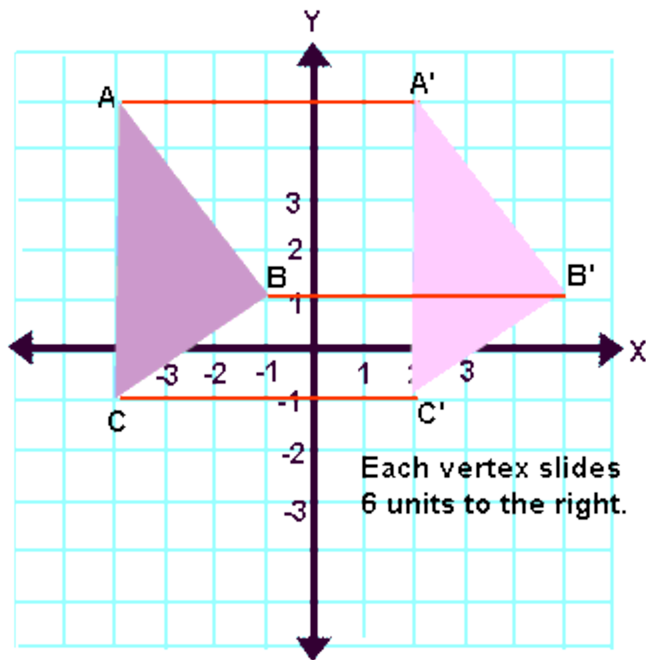


Rotational Symmetry

The figure is rotated around a central point.

Translations

A translation "slides" an object a fixed distance in a given direction. The original object and its translation have the same shape and size, and they face in the same direction. A translation creates a figure that is congruent with the original figure and preserves distance (length) and orientation (lettering order). A translation is a direct isometry.



Translations T (x,y)

Translations

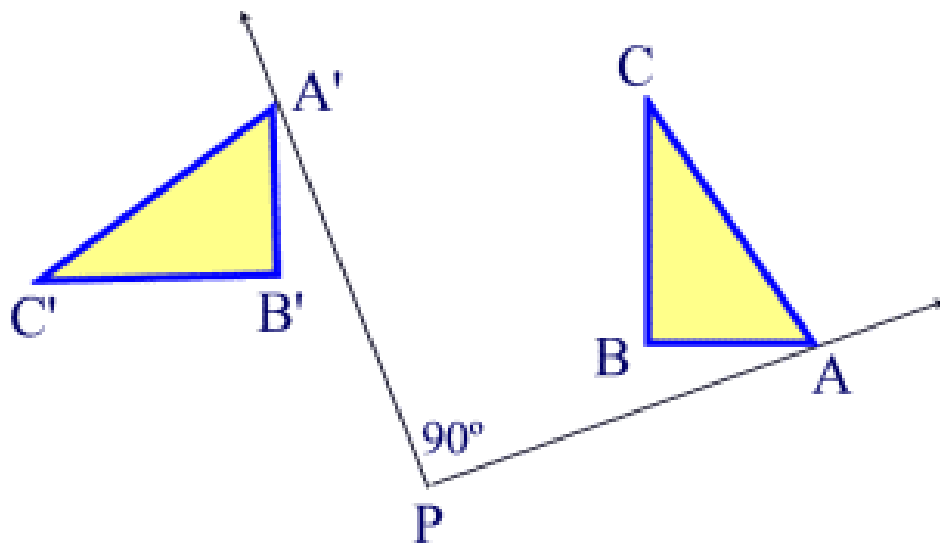
A **translation** "slides" an object a fixed distance in a given direction. The original object and its translation have the **same shape and size**, and they **face in the same direction**. It is a **direct isometry**.

Translation of **h, k:**

$$T_{h,k}(x,y) = (x + h, y + k)$$

Rotations

- A **rotation** is a transformation that turns a figure about a fixed point called the **center of rotation**. Rays drawn from the center of rotation to a point and its image form an angle called the **angle of rotation**.
(notation R_{degrees})



Rotations $R(x,y)$

Rotations

(assuming center of rotation to be the origin)

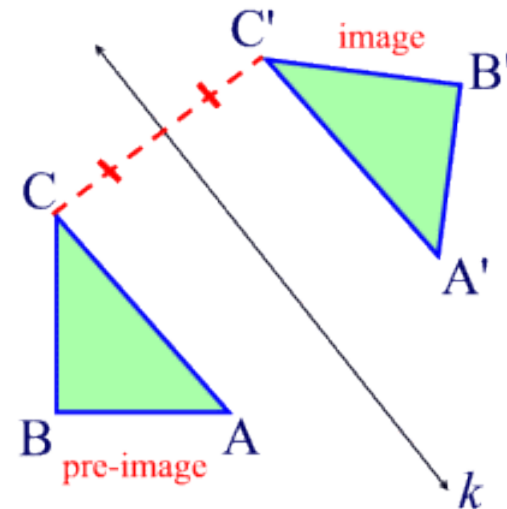
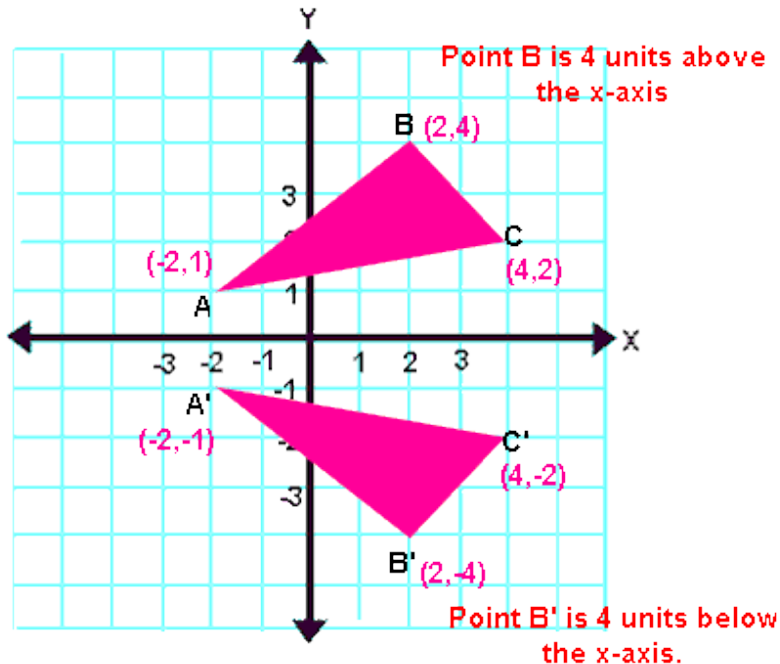
A **rotation** turns a figure through an angle about a fixed point called the center. A **positive angle** of rotation turns the figure **counterclockwise**, and a **negative angle** of rotation turns the figure in a **clockwise direction**. It is a **direct isometry**.

Rotation of 90° :	$R_{90^\circ}(x,y) = (-y,x)$
Rotation of 180° :	$R_{180^\circ}(x,y) = (-x,-y)$ (same as point reflection in origin)
Rotation of 270° :	$R_{270^\circ}(x,y) = (y,-x)$

Reflections

- A **reflection** over a line k (notation r_k) is a transformation in which each point of the original figure (pre-image) has an image that is the same distance from the line of reflection as the original point but is on the opposite side of the line. Remember that a reflection is a flip. Under a reflection, the figure does not change size.

The line of reflection is the perpendicular bisector of the segment joining every point and its image.



Reflections $r(x,y)$

Line Reflections

A reflection is a **flip**. It is an **opposite isometry** - the image does not change size but the lettering is reversed.

Reflection in the x-axis:	When you reflect a point across the x -axis, the x -coordinate remains the same, but the y -coordinate is transformed into its opposite. $P(x, y) \rightarrow P'(x, -y)$ or $r_{x\text{-axis}}(x, y) = (x, -y)$
Reflection in the y-axis:	When you reflect a point across the y -axis, the y -coordinate remains the same, but the x -coordinate is transformed into its opposite. $P(x, y) \rightarrow P'(-x, y)$ or $r_{y\text{-axis}}(x, y) = (-x, y)$
Reflection in $y = x$:	When you reflect a point across the line $y = x$, the x -coordinate and the y -coordinate change places. $P(x, y) \rightarrow P'(y, x)$ or $r_{y=x}(x, y) = (y, x)$
Reflection in $y = -x$:	When you reflect a point across the line $y = -x$, the x -coordinate and the y -coordinate change places and are negated (the signs are changed). $P(x, y) \rightarrow P'(-y, -x)$ or $r_{y=-x}(x, y) = (-y, -x)$

Point Reflections

A **point reflection** exists when a figure is built around a single point called the center of the figure. It is a **direct isometry**.

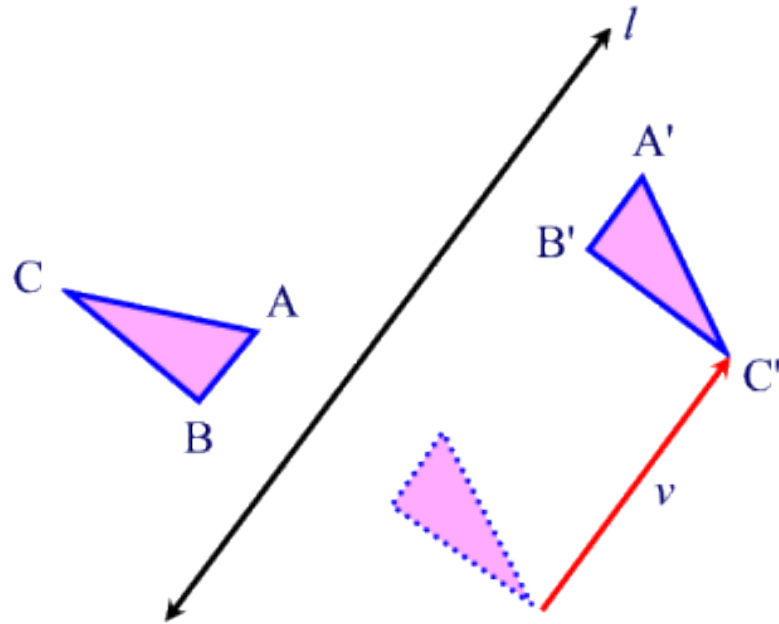
Reflection in the **Origin:**

While any point in the coordinate plane may be used as a point of reflection, the most commonly used point is the origin.

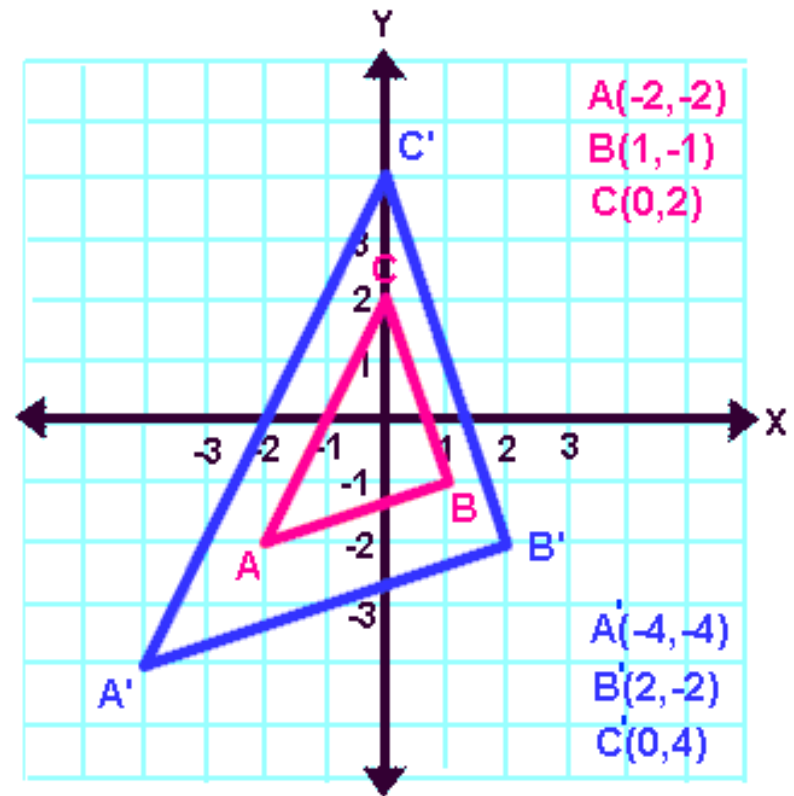
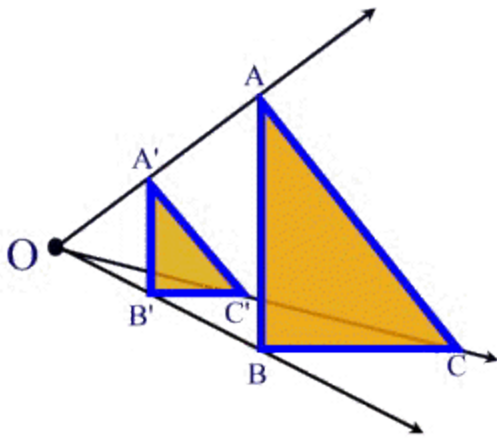
$$P(x, y) \rightarrow P'(-x, -y) \quad \text{or} \quad r_{\text{origin}}(x, y) = (-x, -y)$$

Glide Reflections

Reflection over a line + translation



Dilations



Dilations $D_k(x,y)$

Dilations

A **dilation** is a transformation that produces an image that is the **same shape** as the original, but is a **different size**. **NOT an isometry**. Forms similar figures.

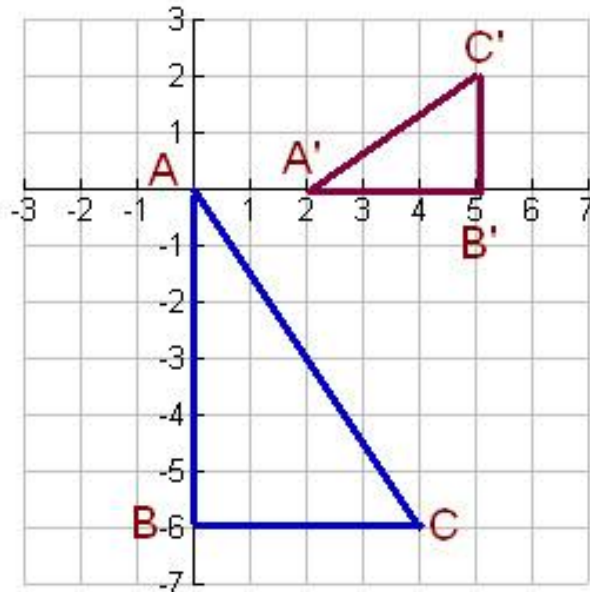
Dilation of scale factor **k**:

The center of the dilation is assumed to be the origin unless otherwise specified.
 $D_k(x, y) = (kx, ky)$

Compositions-

Done in order from right to left

6. Which composition of transformations will map $\triangle ABC \rightarrow \triangle A'B'C'$?



Choose: (dilations centered at origin)

- $R_{90^\circ} \circ D_{0.5} \circ T_{2,0}$
- $T_{2,0} \circ R_{90^\circ} \circ D_{0.5}$
- $R_{90^\circ} \circ D_2 \circ T_{2,0}$
- $T_{2,0} \circ R_{90^\circ} \circ D_2$

Analytic Geometry

Slopes and Equations:

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y = mx + b \text{ slope-intercept}$$

$$y - y_1 = m(x - x_1) \text{ point-slope}$$

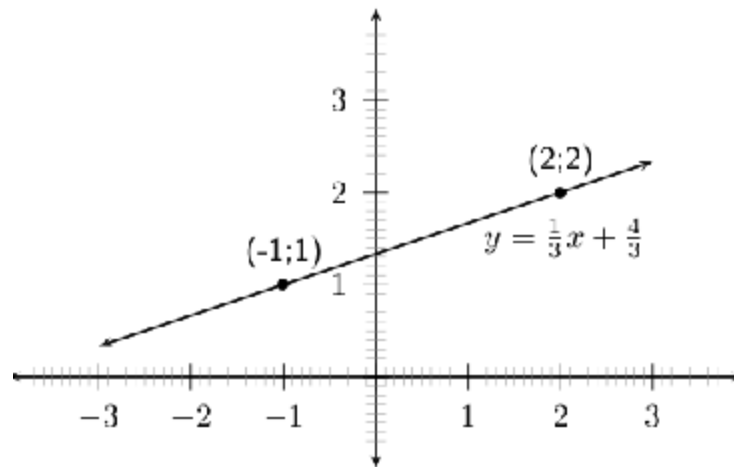
Coordinate Geometry Formulas:

Distance Formula:

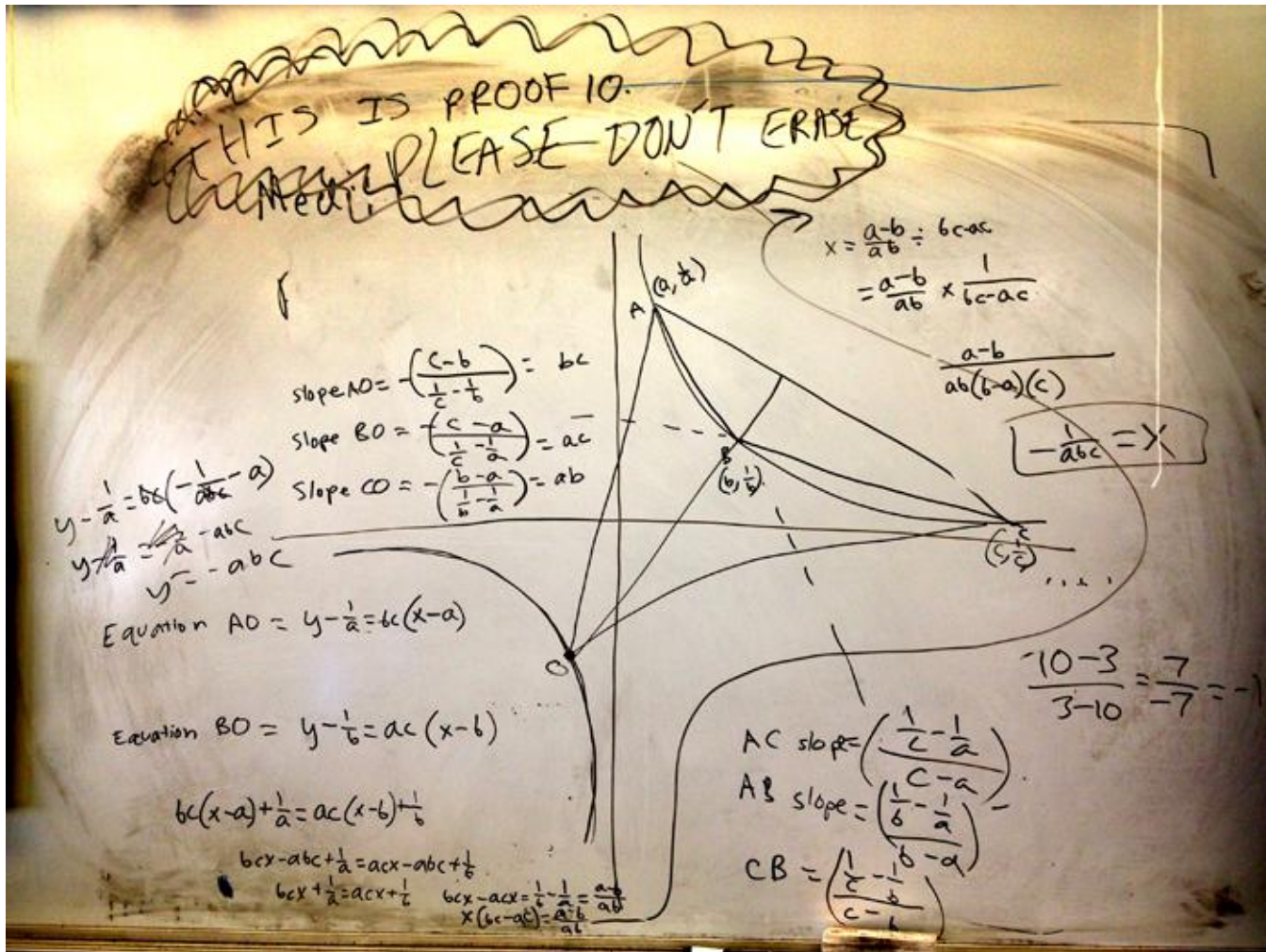
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



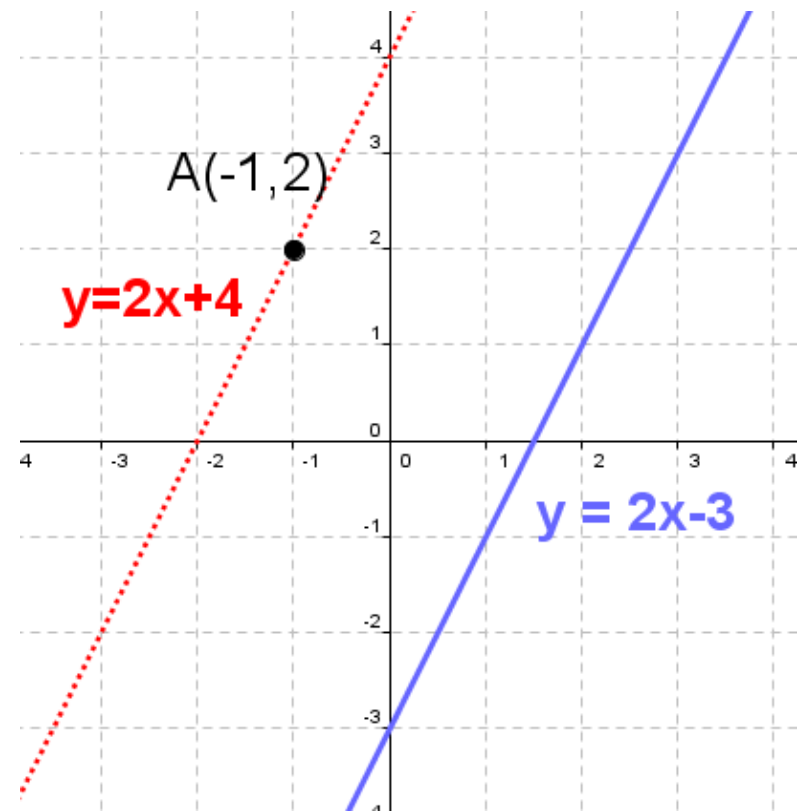
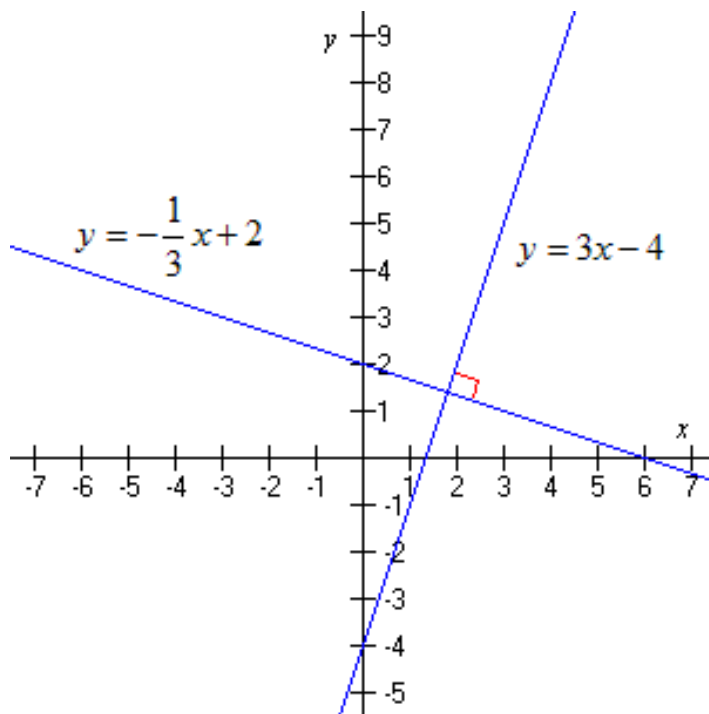
Analytic Geometry



Analytic Geometry

The product of slopes of perpendicular lines is -1

If two lines have the same slope, they are parallel



Circle Equations

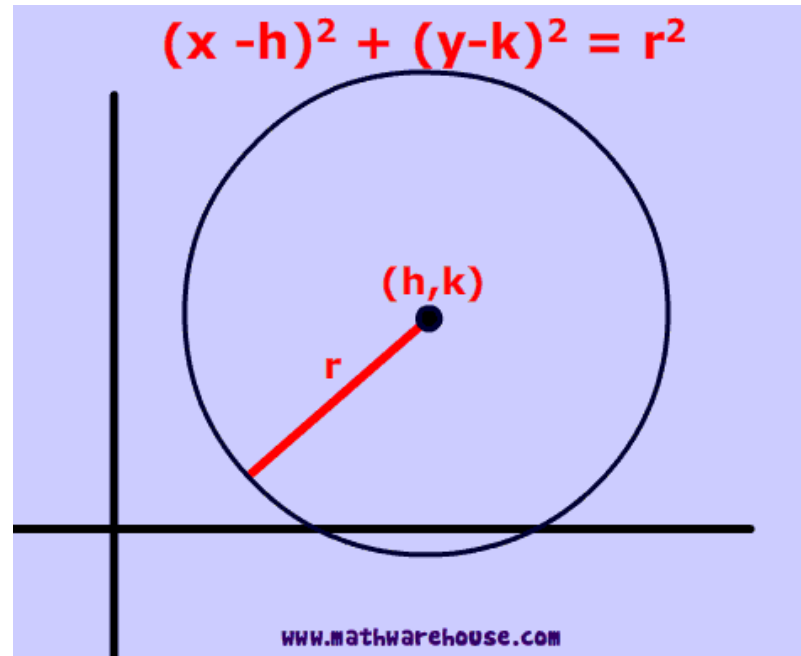
Circles:

Equation of circle center at origin:

$$x^2 + y^2 = r^2 \quad \text{where } r \text{ is the radius.}$$

Equation of circle not at origin:

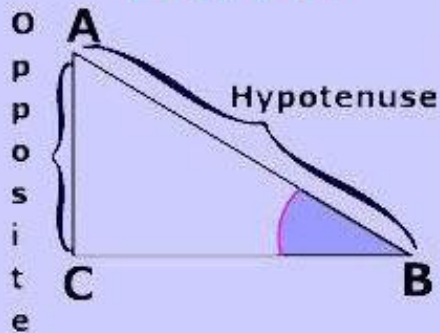
$(x - h)^2 + (y - k)^2 = r^2$ where (h, k) is the center and r is the radius.



Trigonometry

The **Sine**, **Cosine** and **Tangent** functions express the **ratios** of sides of a **right triangle**.

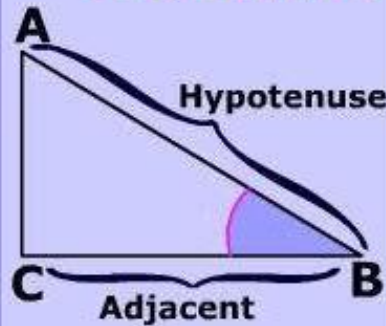
Sine



$$\frac{\text{opposite}}{\text{hypotenuse}}$$

SOH

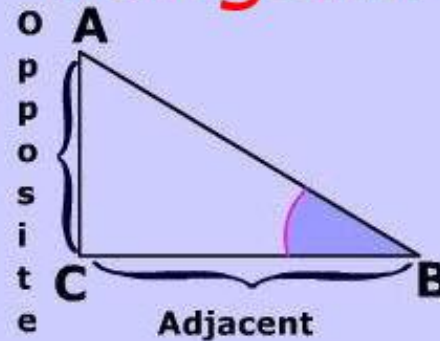
Cosine



$$\frac{\text{adjacent}}{\text{hypotenuse}}$$

CAH

Tangent



$$\frac{\text{opposite}}{\text{adjacent}}$$

TOA

Trigonometry

$$\sin A = \cos(90-A)$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

This well-known equation is called a **Pythagorean Identity**.

The value of θ is immaterial.

Law of Cosines

If a problem refers to
3 sides and 1 angle,
use Law of Cosines.

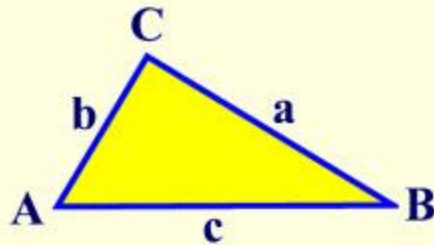
Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

In triangle problems dealing with **2 sides and 2 angles** we have seen that the **Law of Sines** is used to find the missing item. There are many problems, however, that deal with **all three sides and only one angle** of the triangle. For these problems we have another method of solution called the **Law of Cosines**.

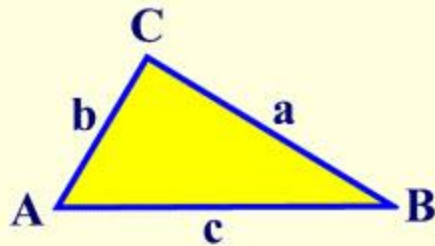


With the diagram labeled at the left,
the Law of Cosines is as follows:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Notice that $\angle C$ and side c are at opposite ends of the formula. Also, notice the resemblance (in the beginning of the formula) to the Pythagorean Theorem.

Law of Sines



In this diagram, notice how the triangle is labeled. The capital letters for the vertices are repeated in small case on the side opposite the corresponding vertex.

side a is opposite $\angle A$
side b is opposite $\angle B$
side c is opposite $\angle C$

working together as
partners!

The ratios of each side to the sine of its "partner" are equal to each other.

**If a problem refers to
2 sides and 2 angles,
use the Law of Sines.**

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Missing Lines

- The key to many problems is drawing the magic missing line
- Segments that stop inside figures should be extended
- Label lengths as you find them
- If you see 30, 60, or 90 degree angles- buildm 30-60-90 degree triangles
- When in doubt, build right triangles

Good Luck!

